A POPULAR INTRODUCTION TO QUANTUM GRAVITY: WHY WE STILL CAN'T MODEL OUR UNIVERSE?

Gabriele Gionti, S.J.

SPECOLA VATICANA
VATICAN OBSERVATORY

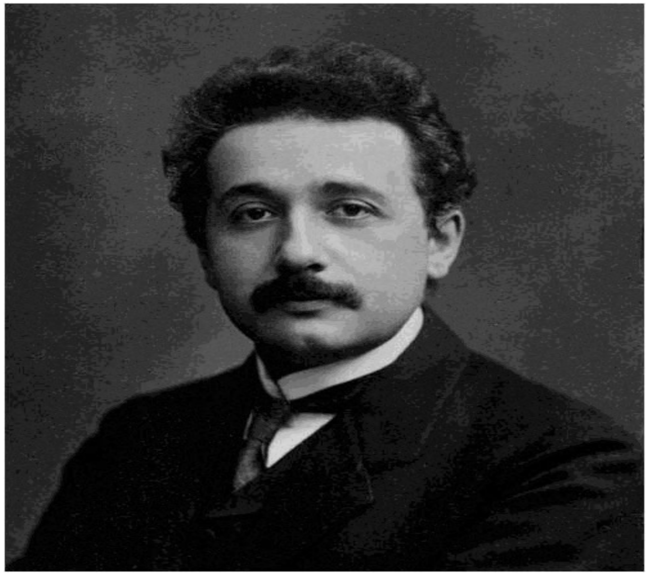
INFN
Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali di Frascati

ASTRONOMICAL OBSERVATORY OF TERAMO

TERAMO, February 05th , 2026

1915

"Annus Mirabilis" !

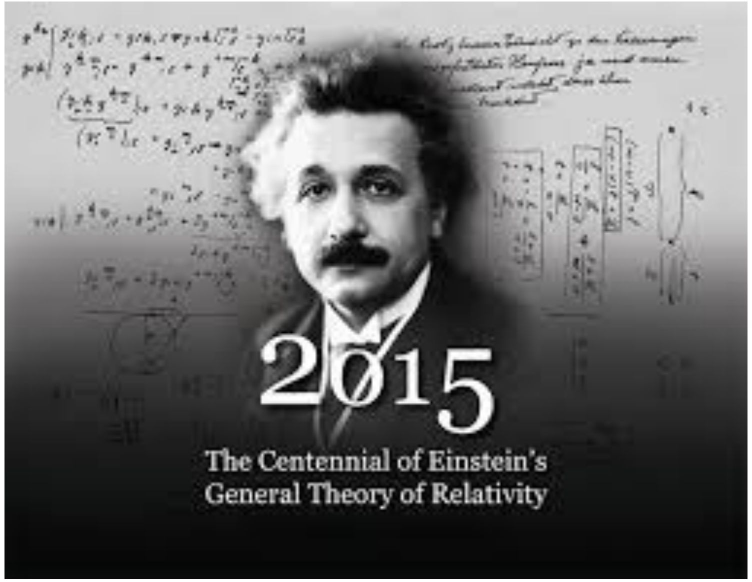


Albert Einstein:
Equations of the Gravitational Field

David Hilbert:
Action of the Gravitational Field

Emmy Noether:
Conservations Laws

GENERAL RELATIVITY



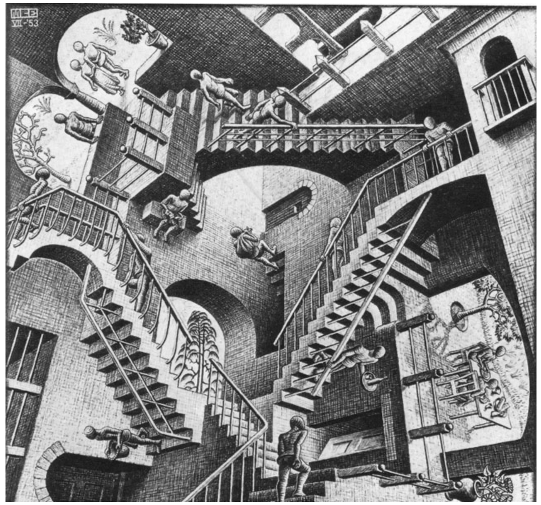
- "Extension" of Special Relativity.

- Deal with "non-inertial frames"
  (case in which the frame "relative" velocity
  is not constant).

- One fundamental principle:

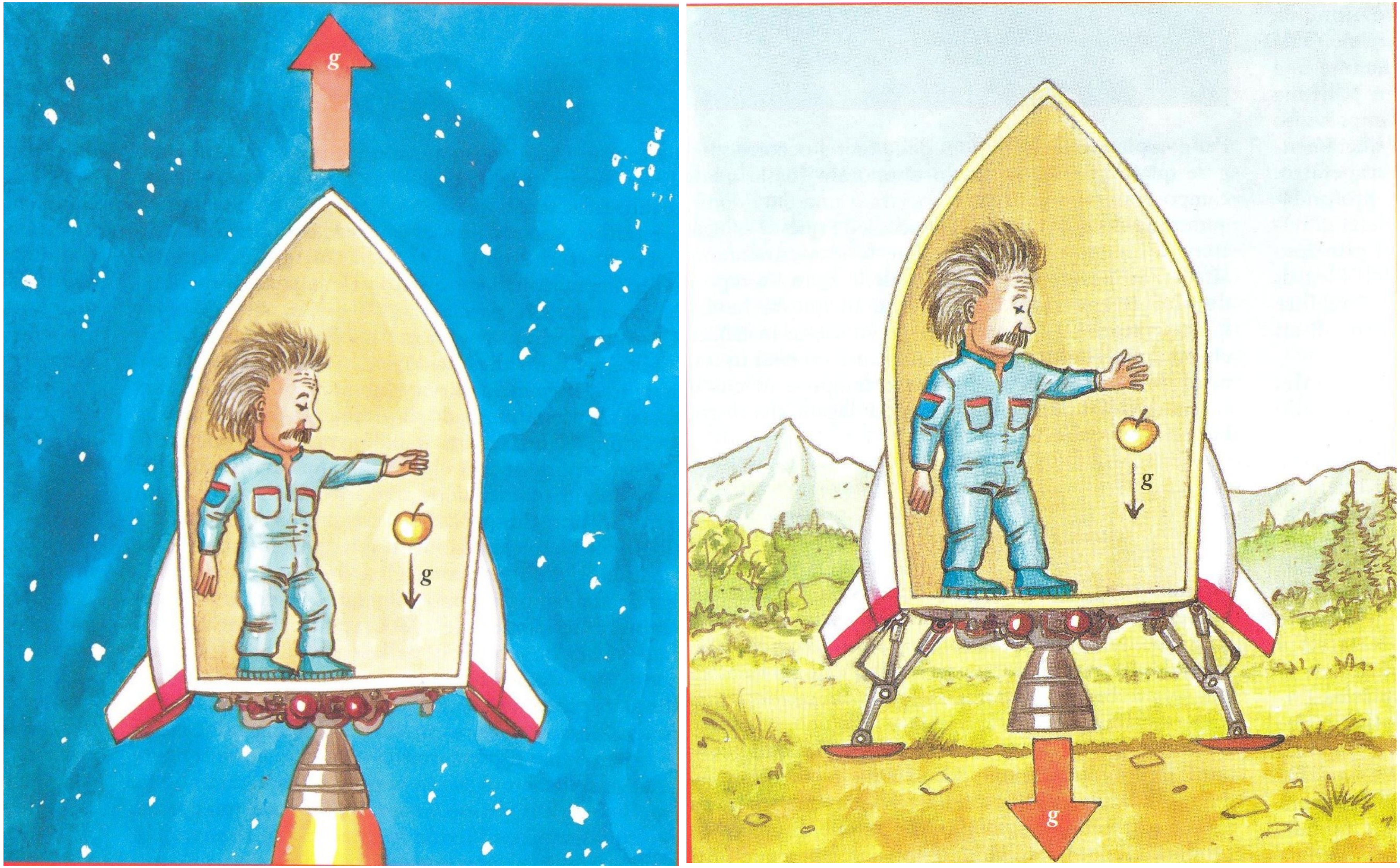
The principle of equivalence "the mass equivalence"

$$m_g = m_i$$



- The gravitational effects are locally indistinguishable from the acceleration effects.

Gravity and Acceleration



Gravity and Acceleration

An astronaut will experience no weight in a free-falling starship in the empty space. The free-fallisng starship cancels the gravitational effects.

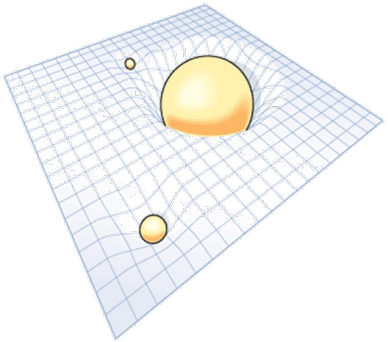
GENERAL RELATIVITY

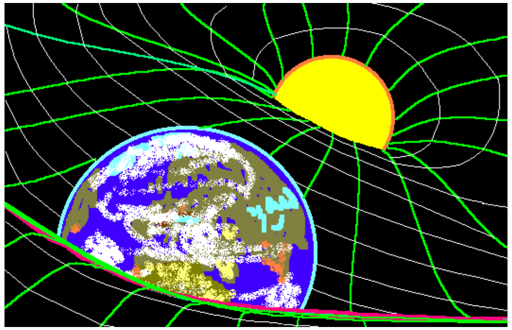
- Corollary of the Equivalence Principle
- The "Covariance Principle": the Laws of Physics are the same (covariant) in every reference frame! (therefore also non-inertial..)

GENERAL RELATIVITY

- The Space-Time is a "physical entity"; it is a dynamical entity that is modified by the presence of massive bodies (as well as Energy).

- The massive bodies modify Space-Time, which gains curvature.





- Free falling bodies move on the analogous of straight lines, in inertial reference frames, which are called geodesic lines.

- The gravitational force is not, anymore, an action at distance; it is a field theory like in electromagnetism.

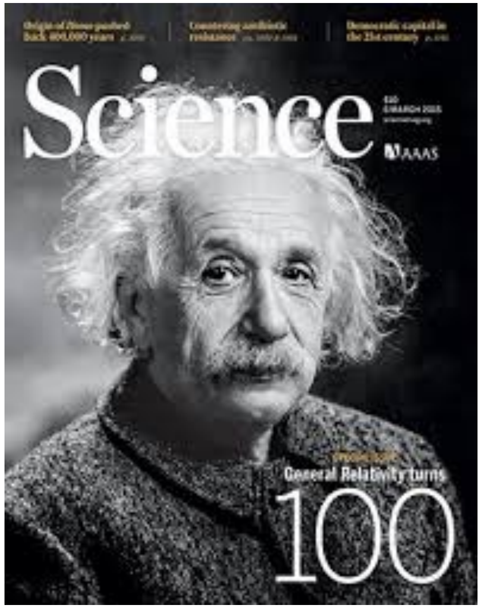
EINSTEIN-HILBERT ACTION AND EINSTEIN'S EQUATIONS

- The Einstein-Hilbert Action Functional is

$$S_{E-H} = \frac{1}{16\pi G} \int_{(M,g)} d^4x \sqrt{-det(g)}\,(R - 2\Lambda + 16\pi G L_m) + \frac{1}{8\pi G} \int_{(\partial M,h)} \sqrt{det(h)} d^3x K$$

- Varying this action, we get Einstein's Field Equations

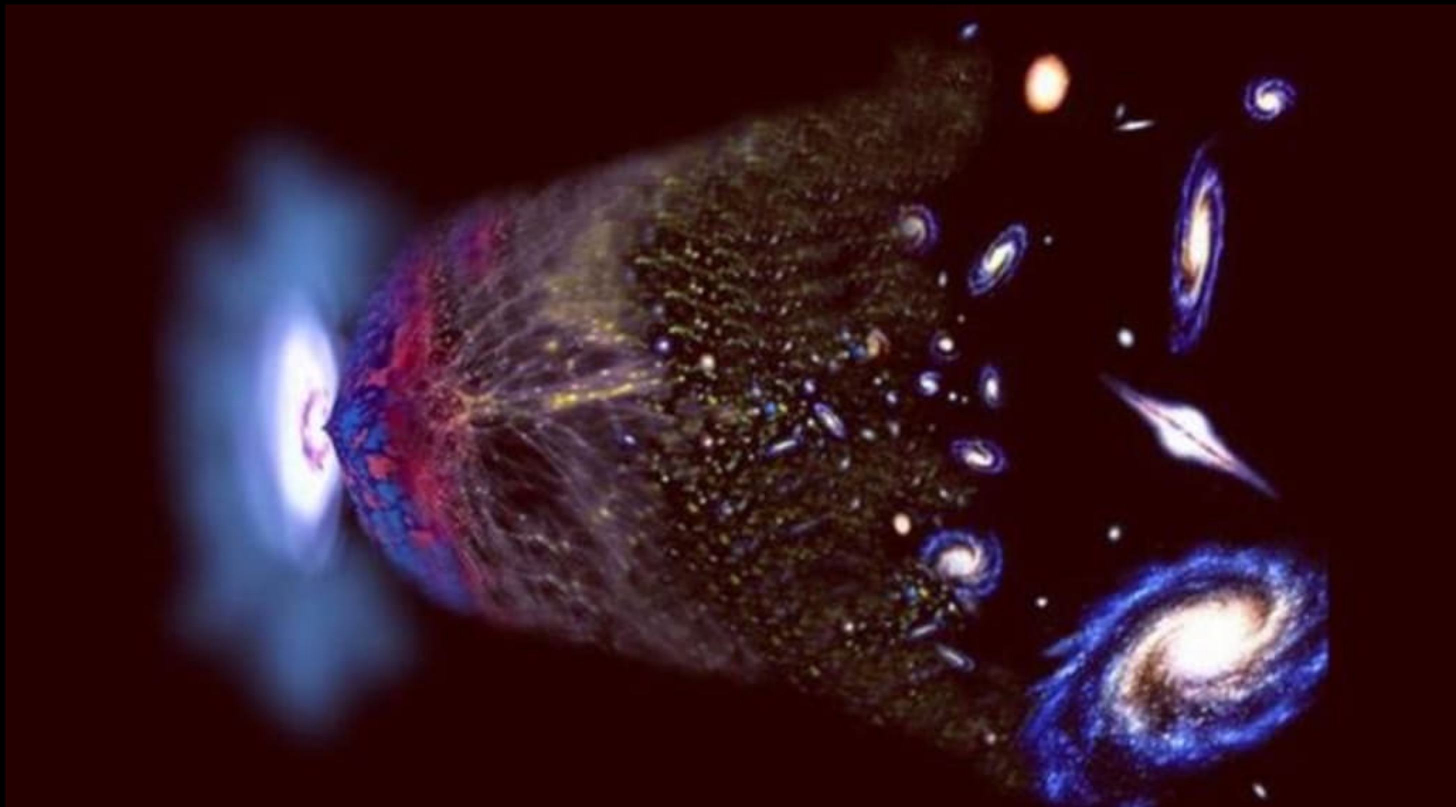
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$



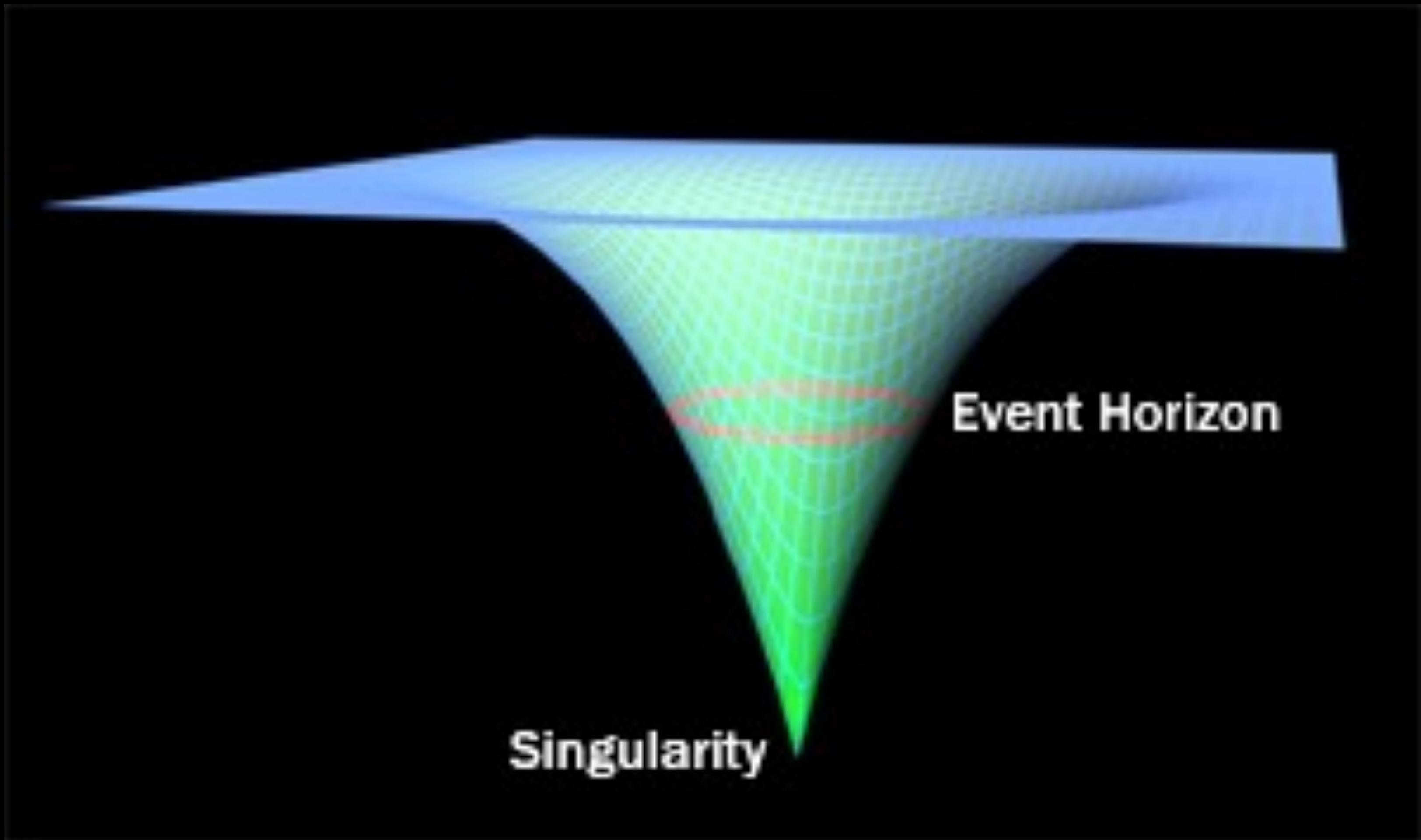
- Ten equations, four first class constraints.

- Two independent Equations.

SINGULARITY IN EINSTEIN'S GENERAL RELATIVITY: BIG-BANG SINGULARITY



SINGULARITY IN EINSTEIN'S GENERAL RELATIVITY: BLACK-HOLE SINGULARITY



Why Quantum Gravity?

$$\text{Planck Temperature} = \sqrt{\frac{\hbar c^5}{G k_B^2}} = 1.42\times10^{32}\,\text{K}$$

$$\text{Planck Mass} = \sqrt{\frac{\hbar c}{G}} = 2.2\times10^{-8}\,\text{kg}$$

$$\text{Planck Time} = \sqrt{\frac{G\hbar}{c^5}} = 5.4\times10^{-44}\,\text{s}$$

$$\text{Planck Length} = \sqrt{\frac{G\hbar}{c^3}} = 1.6\times10^{-35}\,\text{m}$$

$$\text{Planck Energy} = \sqrt{\frac{\hbar c^5}{G}} = 1.22\times10^{19}\,\text{GeV}$$

$$\text{Planck Density} = \frac{c^5}{G^2\hbar} = 5.16\times10^{93}\,\text{g/cm}^2$$

- **All fundamental interactions have a quantum field theory behavior, then there could emerge a contradiction when we couple them to Gravity**

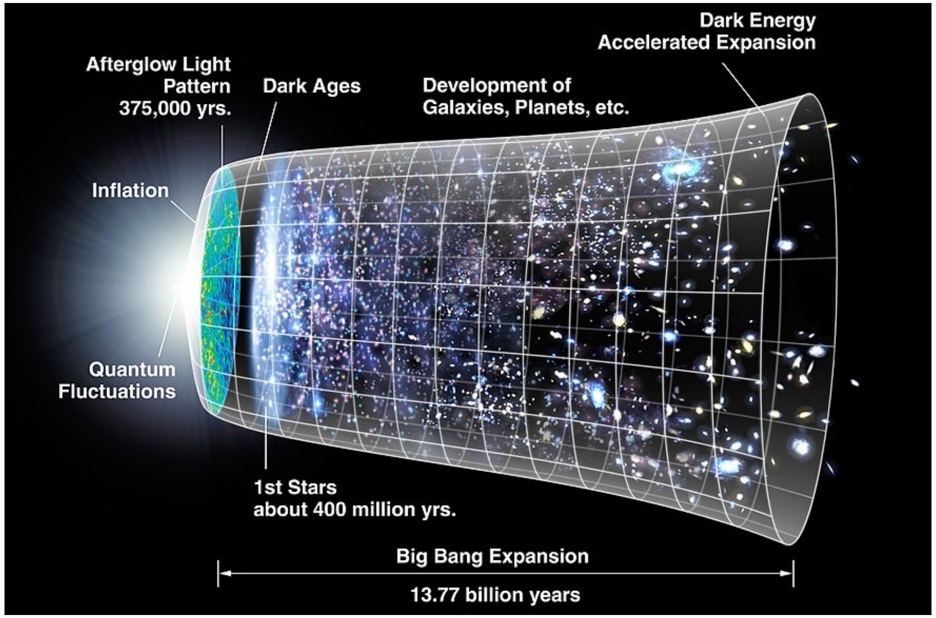
QUANTUM GRAVITY

- Einstein General Relativity is considered succesful phenomenological theory at laboratory, solar system, galatic, for lenght scales $l \gg l_{Pl}$ (=Planck lenght)$\equiv {}^{1}/_{\sqrt{G}} \approx 10^{-33}$cm

- Singularity problem and the quantum mechanical behaviour of matter-energy at small distance suggest a quantum mechanical behaviour of the gravitational field (Quantum Gravity) at small distances (High Energy).

- Many different approaches to Quantum Gravity: String Theory, Loop Quantum Gravity, Non-commutative Geometry, Dynamical Triangulations, Causal Dynamical Triangulations, Asymptotic Safety etc.

- General Relativity is considered an effective theory. It is not pertubatively renormalizable (the Newton constant G has a (lenght)$^{-2}$ dimension)



QUANTUM MECHANICS

- Classical mechanics: $L(q^i(t), \dot{q}^i(t)) = T - U$
- Legendre map: $p_i = \dfrac{\partial L}{\partial \dot{q}^i}$
- Hamiltonian $H(q^i(t), p_i(t)) = T + U$
- Quantization: you need to have a Hamiltonian funtion, written in Cartesian coordinates, and Poisson brackets

$$\{f(q,p), g(q,p)\} = \left\{ \frac{\partial f}{\partial q^i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q^i} \right\} \qquad \{q^i, p_j\} = \delta_i^j ; \{q_i, q_j\} = \{p_i, p_j\} = 0$$

QUANTUM MECHANICS

- Position q^i and momentum p_i become operators on the Hilbert space defined by the wave functions $\psi(q^i, t)$, so that the "Dirac's map" is

$$p_i \mapsto \frac{\hbar}{i}\frac{\partial}{\partial x^i}; \quad q^i \mapsto q^i \qquad p_i\psi(q^i,t) = \frac{\hbar}{i}\frac{\partial}{\partial q^i}\psi(q^i,t) \ ; \ q^i\psi(q^i,t) = q^i\psi(q^i,t);$$

- The Poisson Brackets become commutators among operators on the Hilbert Space

$$\{,\} \mapsto [F,G] \equiv [F,G]\psi = (FG - GF)\psi$$

$$[q^i, p_j]\psi = \delta^i_j\psi; \quad [q^i, q^j]\psi = [p_i, p_j]\psi = 0$$

QUANTUM MECHANICS

- Equation to solve is the Eigenvalue equation

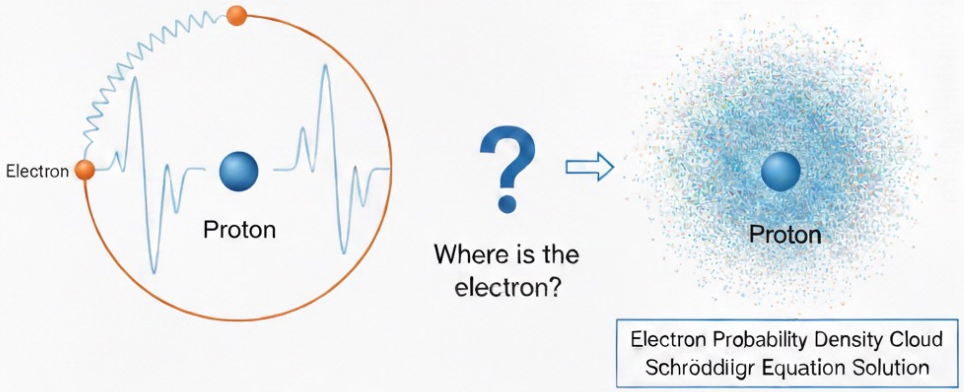
$$H\psi = E\psi$$

- Schrodinger Equation

$$-i\hbar\frac{\partial\psi(q^i,t)}{\partial t} = H\psi \qquad H = -\frac{-\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial {q^i}^2} + U(q^i)$$

- In the case of the Hydrogen Atom the energy eigenvalues are discrete.

Dual Nature the Electron

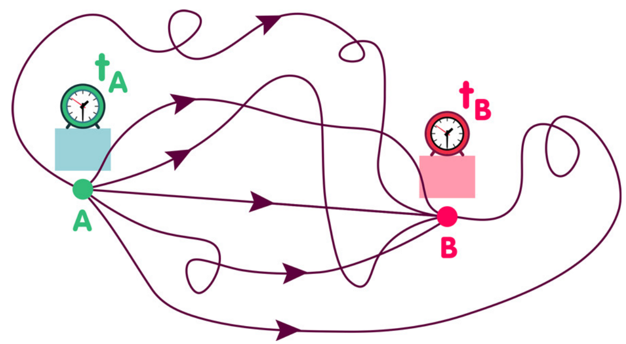


January 28, 2026

FEYNMAN'S PATH INTEGRAL

An equivalent way of "quantizing" is through the Feynman Path Integral. A functional integral over all possible paths, in the "canonical space", is proportional to the matrix element M_{fi} whose "square" gives the probability of transition between two states.

- Feynman's propagator



$$K(q_2, t_2; q_1, t_1) = \int_{q(t_1)=q_1}^{q(t_2)=q_2} [dq(t)] \int [dp(t)]$$

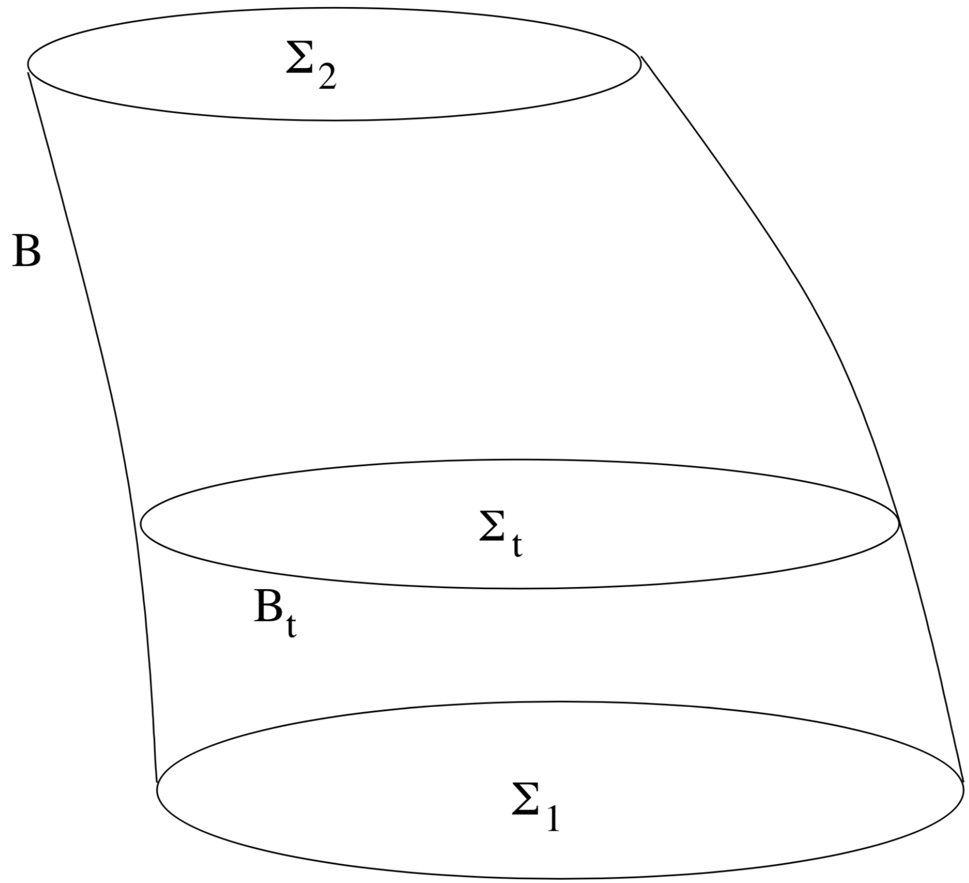
$$\cdot \exp\left\{ \frac{i}{\hbar} \int_{t_1}^{t_2} dt \left[p(t)\dot{q}(t) - H(q(t), p(t)) \right] \right\}$$

- Connection with the Wave Function

$$\langle x' t | \psi \rangle = \psi(x', t) = \int \langle x', t | x'', 0 \rangle dx'' \langle x'', 0 | \psi \rangle$$

$$= \int dx'' \, K(x', t; x'', 0) \psi(x'', 0)$$

CANONICAL QUANTUM GRAVITY
(non-perturbative)



CANONICAL QUANTUM GRAVITY

It is one of the oldest approach in Quantum Gravity

It is based on ADM (Arnowit, Deser and Misner) technique: a three-dimensional space-like surface evolving in time.

The goal is to write a Hamiltonian density functional of the gravitational field.

This can be done and one can write the Wheeler-DeWitt equation for the wave-function of the Universe.

It is difficult to solve except in very particular cases called "mini-superspace" models of Quantum Cosmology.

CANONICAL QUANTUM GRAVITY

- Hamiltonian Density of the Gravitational Field (Hamiltonian Constraint)

$$\mathcal{H} = 16\pi G \frac{1}{\sqrt{q}}(\pi_{ij}\pi^{ij} - \frac{1}{2}\pi^2) - \frac{1}{16\pi G}\sqrt{q}\,({}^{(3)}R - 2\Lambda)$$

- Momenta (Momenta Constraints)

$$H_i = -2\sqrt{q}\nabla_j\left(\frac{\pi_i^j}{\sqrt{q}}\right)$$

- Quantum Commutation Relation (q_{ij} is a three-metric on constant time three-surfaces)

$$[q_{ij}(x), \pi^{kl}(x')] = \frac{i}{2}(\delta_i^k\delta_j^l + \delta_i^l\delta_j^k)\tilde{\delta}^3(x - x'),$$

$$\pi^{kl} = -i\frac{\delta}{\delta q_{kl}}.$$

- Wheeler-DeWitt Equation

$$\left[(16\pi G)G_{ijkl}\frac{\delta}{\delta q_{ij}}\frac{\delta}{\delta q_{kl}} + \frac{1}{16\pi G}\sqrt{q}\,({}^{(3)}R - 2\Lambda)\right]\Psi[q] = 0, \quad H_i\Psi[q] = 0;$$

$$G_{ijkl} = \frac{1}{2}q^{-1/2}\left(q_{ik}q_{jl} + q_{il}q_{jk} - q_{ij}q_{kl}\right)$$

CANONICAL QUANTUM GRAVITY

- Hartle-Hawking proposal: no-boundary-boundary proposal. Solution of Wheeler-DeWitt equation for FLRW minusuperspace model

- Flat FLRW metric

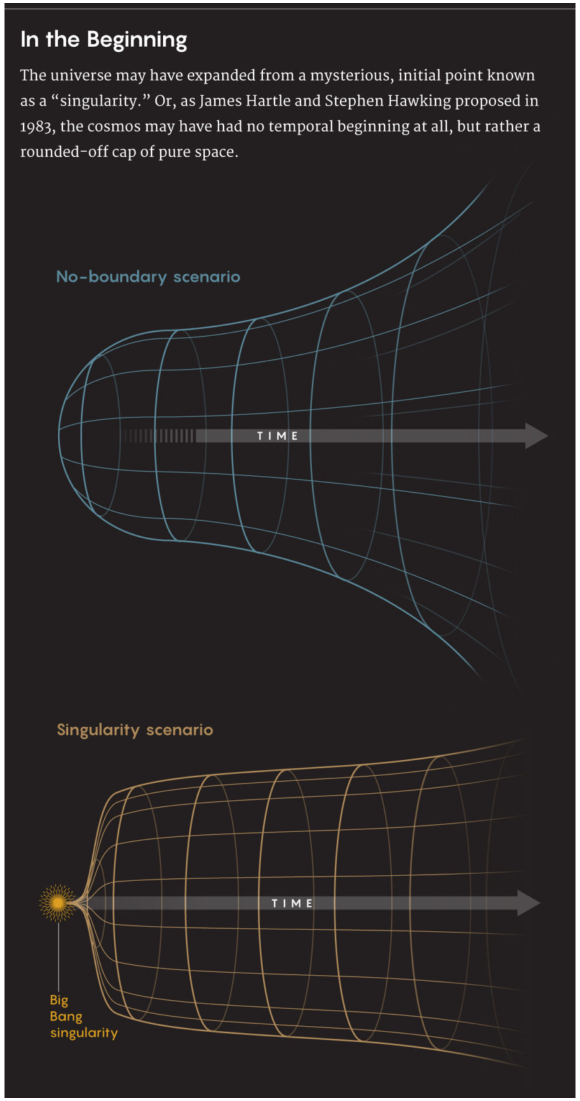
$$ds^2 = dt^2 - R^2(t)d\Omega_3^2$$

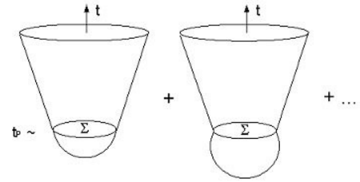
- WDW equation in FLRW Minisuperspace

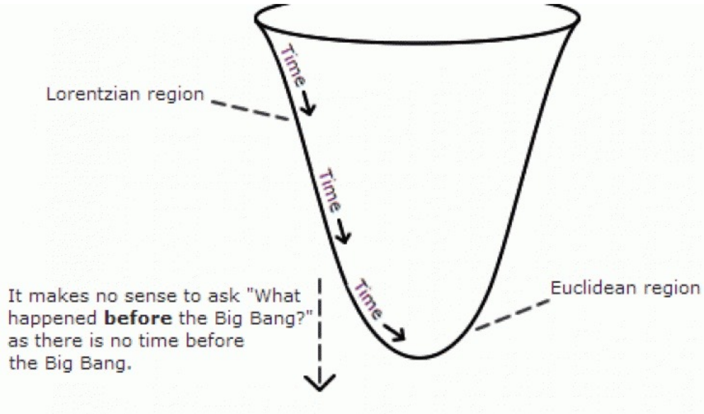
$$\left[\frac{\partial^2}{\partial R^2} - \frac{9\pi^2}{4G^2}\left(R^2 - \frac{\Lambda}{3}R^4\right)\right]\Psi(R) = 0$$

HARTLE-HAWKING SOLUTION

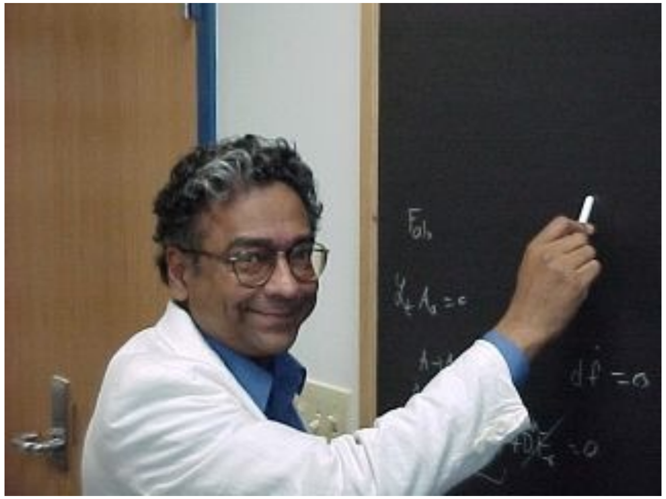
(No-Boundary)-Boundary proposal







LOOP QUANTUM GRAVITY



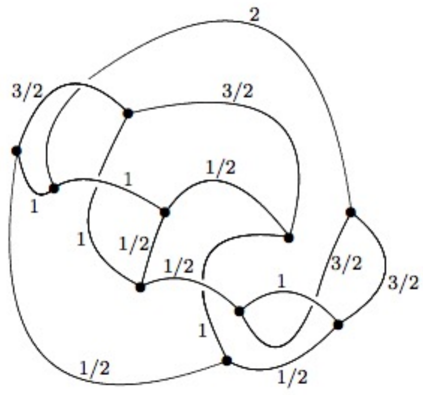
Abhay Ashtekar

Lee Smolin

Carlo Rovelli

LOOP QUANTUM GRAVITY

- Ashtekar's variable for Eistein General Relativity
- They still recover Einsten General Relativity
- The important feature is that Space is made out of Spin Network (Loops)



LOOP QUANTUM GRAVITY

- The next step is to define "holonomy matrices" of this kind on the Spin-Network

For any curve $\gamma : [0,1] \to \Sigma$, consider the holonomy

$$U_\gamma(s_1, s_2) = P \exp\left\{ - \int_{s_1}^{s_2} ds \frac{dx^i(s)}{ds} A_i{}^{\hat{I}} \tau_{\hat{I}} \right\},$$

where P denotes path ordering. Then for a closed curve, the "Wilson loop" $\mathrm{Tr}\, U_\gamma(0,1)$ is gauge invariant. More generally, let Γ be a graph, and define a "coloring" as follows:

- It is possible to prove that "functionals of holonomy matrices" solve the Wheeler-DeWitt equation expressed in Ashtekar's variables

LOOP QUANTUM GRAVITY

The area operator, for example, has eigenvalues of the form

$$A = 8\pi\gamma G \sum_i \sqrt{j_i(j_i+1)},$$

- It follows that the area operator is quantized

- This theory has several mathematical problems (i.e. the algebra of the Hamiltonian constrain does not close).

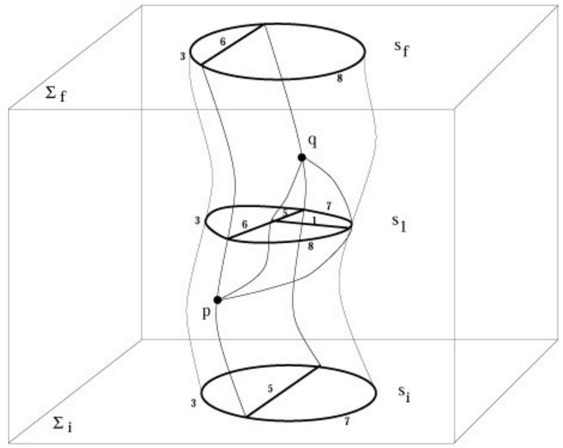
BLACK HOLE ENTROPY

- Bekenstein-Hawking formula:

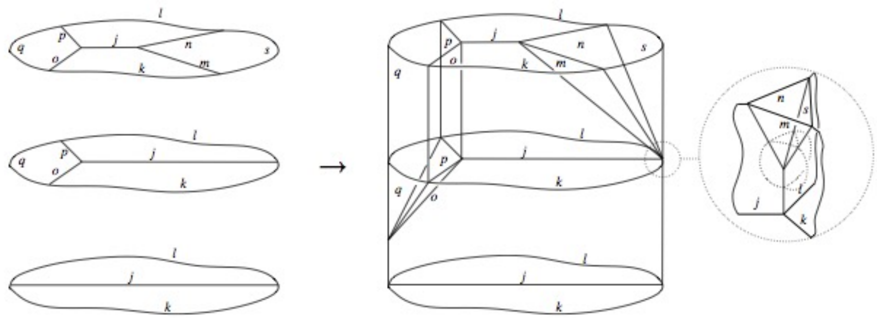
$$S_{BH} = \frac{A}{4L_P^2} = \frac{c^3 A}{4G\hbar}$$

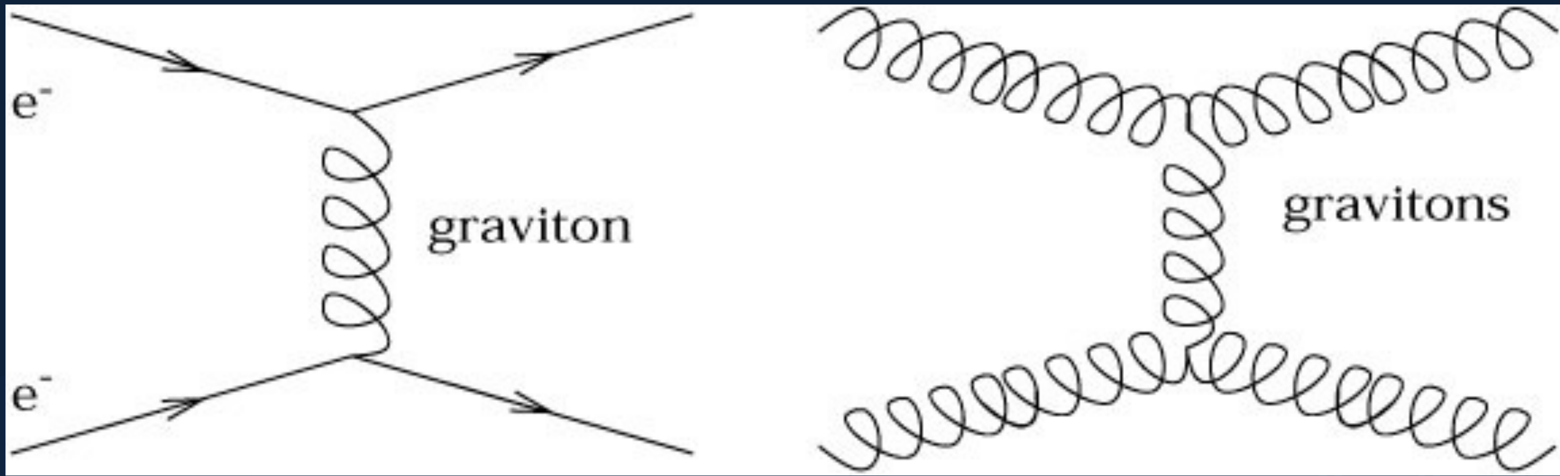
- The same formula can be derived from Loop Quantum Gravity starting with

  S=k ln N(A), and the microstates are Spin networks intersecting the surface A



- Spin Foam (time evolution of Spin Networks)

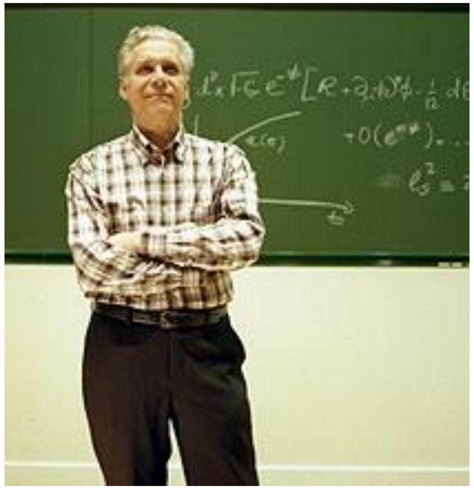




QUANTUM GRAVITY (via Path Integral)

- Quantized G.R. is perturbative non renormalizable. It is believed it is an effective theory valid for $l \gg l_{Pl}$, rather than a fundamental (microscopic) theory valid at arbitrary small distances.
- In general, theories are fundamental if they are perturatively renormalizable, if their infinities can be absorbed by re-defininig only finitely many parameters (m, e, …)
- Perturbative non-renormalizable theories: increasing number of conter terms as the loop order increases. Infinite many parameters, no predictive power

STRING THEORY



- Gabriele Veneziano's formula in dual Model for Strong interactions:
Duality in s and t channel interchange of strong interactions

- Generalizations of Veneziano Model (Nielsen, Nambu, Susskind) and Relativistic String first appearance.

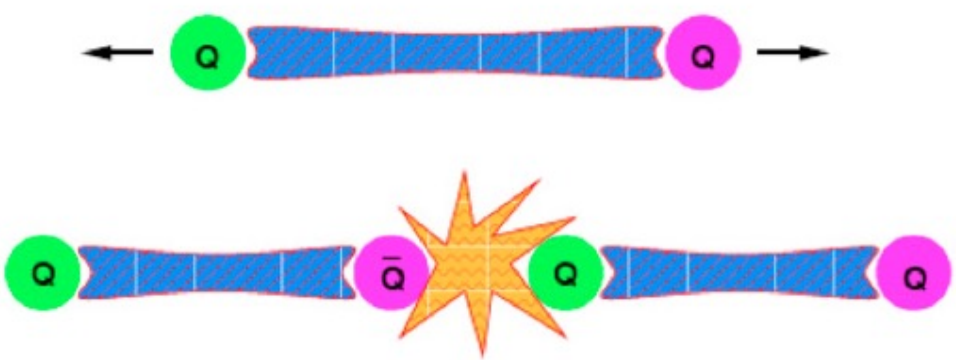
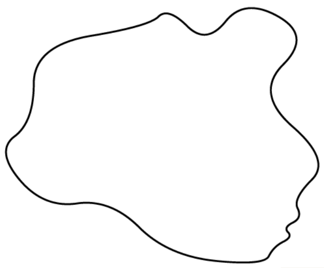
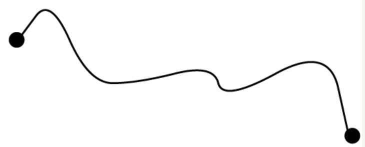


Figure 6: As quarks are pulled apart, eventually new quarks appear.

- Some problem with the model: needed 26 dimensions, states with negative energy(tachyons), massless spin-2 unknown states (later identified as gravitons)

- A SU(3) Gauge theory (QCD) described better strong interactions



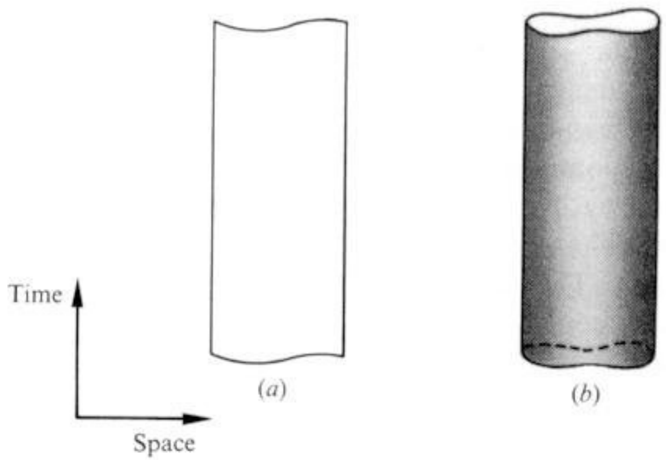


STRING THEORY

Polyakov action for Bosonic strings

$$I_{\rm str} = \frac{1}{4\pi\alpha'} \int_S d^2\sigma \sqrt{g} g^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

World-sheets for (a) open and (b) closed strings.



QUANTIZATION OF BOSONIC STRINGS

- One quantizes Bosonic String like any vector field in Quantum Field Theory (Expand the field in a Fourier base and impose commutation relations among the coefficients like the harmonic oscillator).

- Open and Closed Strings both have a tachyon (negative mass) in their quantum spectrum

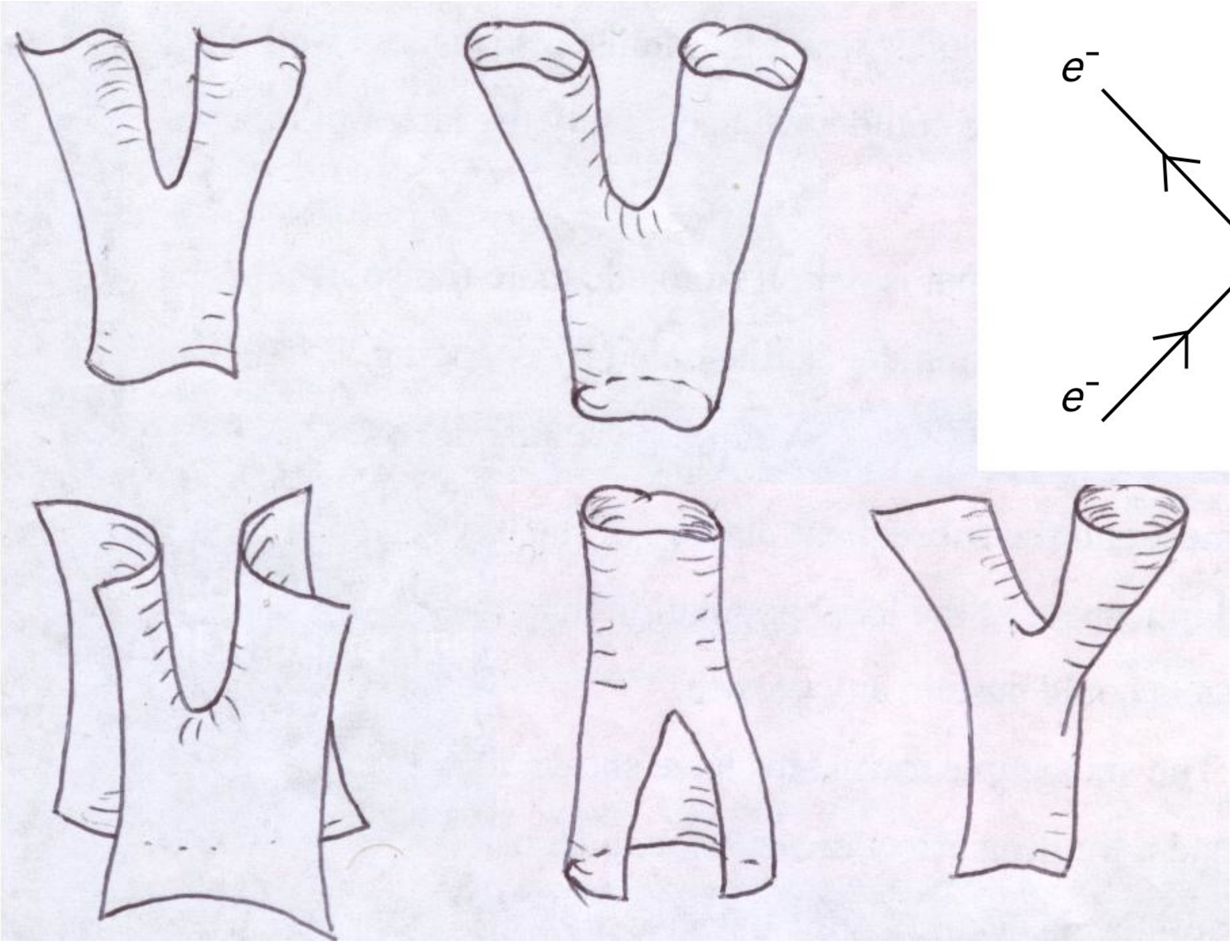
- Closed Strings have a spin-two massless excited state and a massless scalar field called dilaton

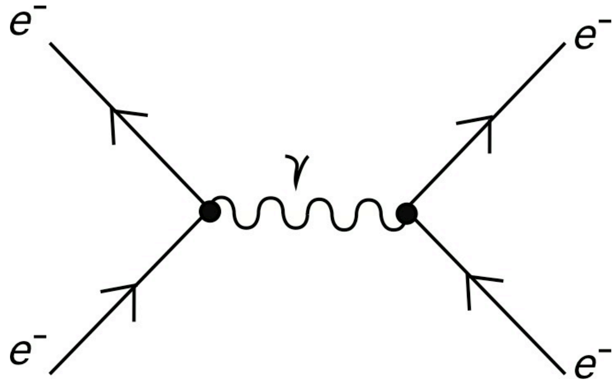
- d=26 for consistency conditions

- The presence of tachyon makes Bosonic String Theory unstable

INTERACTIONS OF BOSONIC STRINGS

- Feynman diagrams of Bosonic String interactions





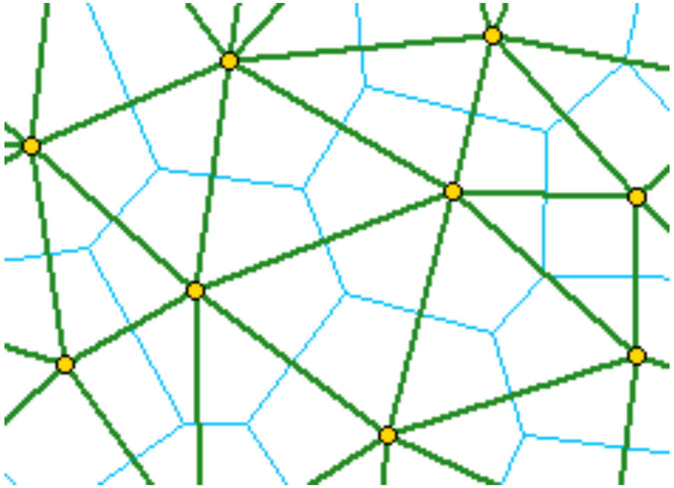
- Bosonic String Theory expansion is finite (one loop… really), no divergences…so perturbative Quantum Gravity looks renormalizable

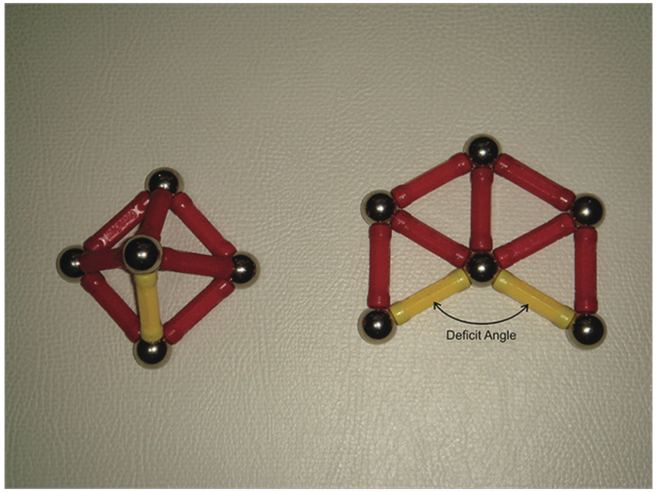
SUPERSTRING THEORY

- Bosonic String Theory is un-stable (tachyons)
- It is possible to introduce a new String Theory such that for every Bosonic degree of freedom $X^m(t,s)$ there corresponds a Fermionic (anti-commuting) degree of freedom $\psi^m(t,s)$.
- The theory does not have tachyons in its spectrum.
- In order to be consistent, it needs ten (9+1) dimensions
- There are five non-equivalent Superstring Theories
- One of the important result in String Theory is that they reproduce Bekestein-Hawking formula for Black-hole Entropy in the Quantum regime

REGGE-CALCULUS

- Triangulated Manifolds (PL-Manifolds)



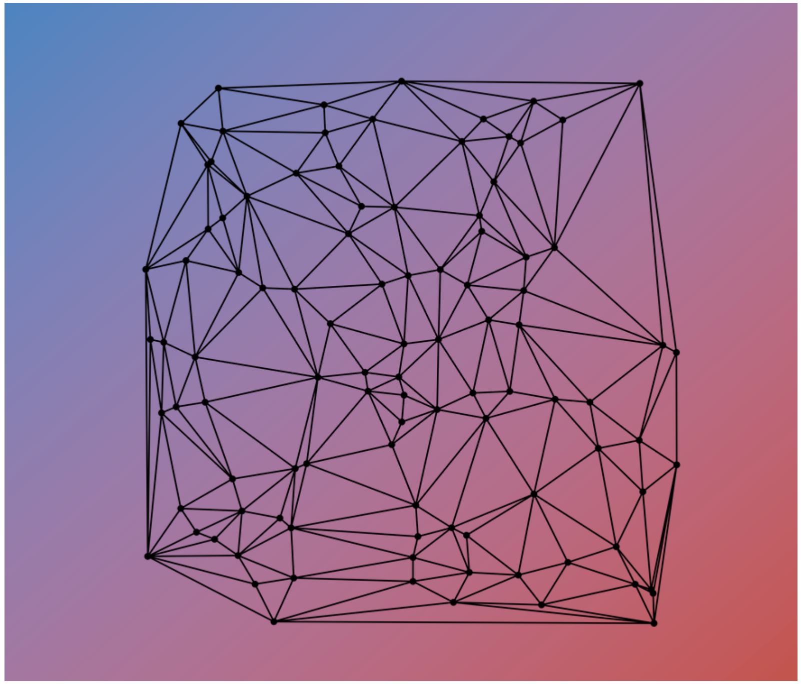


- Curvature (h stands for “hinges”, n-2 simplexes carrying the curvature

$$K(h) = 2\pi - \sum_{\sigma_h^i \supset h} \theta(\sigma_h^i, h)$$

- Einstein-Hilbert Action: $S_R = \sum_h K(h)V(h)$

- Regge calculus convergence to G.R. in Measure



QUANTUM REGGE CALCULUS

- Manifold and triangulation is fixed. One varies over the edge lengths.

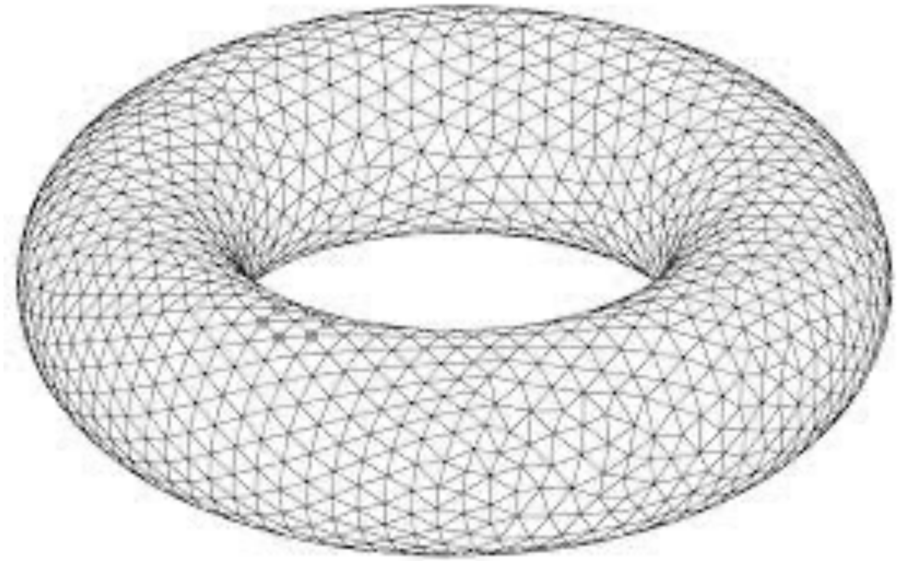
$$Z = \int D[l^2] e^{-S_R[l^2]}$$

- The Quantization is implemented through the Path Integral

$$D[l^2] \propto l^2 dl \ (?)$$

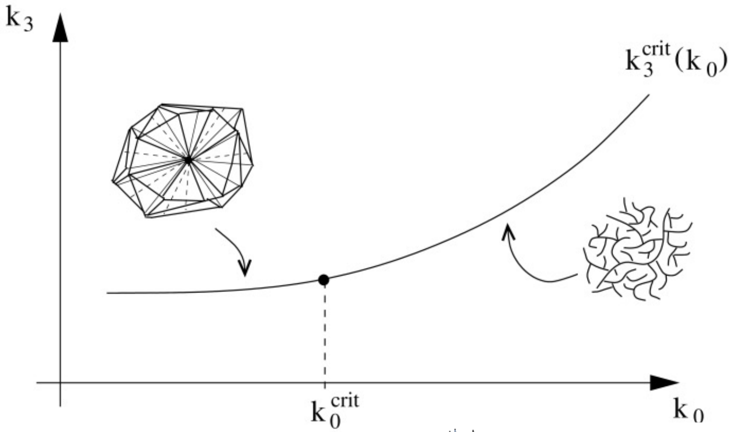
DYNAMICAL TRIANGULATIONS

- Dynamical Triangulations is a variant of Regge Calculus in which the edge lenghts are fixed and you vary the number of Triangles . Quantum Regge path-integral, above, is substituted with sum.

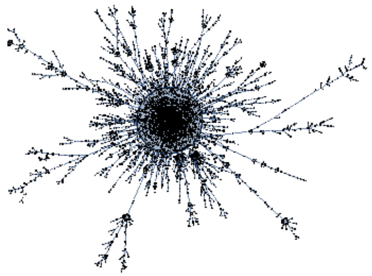


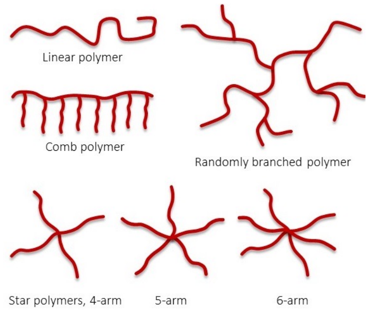
DYNAMICAL TRIANGULATIONS

- Phase diagram



- In general for $k_0 < k_0^{crit}$ Crumpled phase





- $k_0 > k_0^{crit}$, Branched polymer phase

- It is unclear, yet, if there is a second order phase transition.

- If the phase transition is not second order, the discretization does not work

IS THE QUANTUM MEASURE FOR REGGE CALCULUS RIGHT ?

- Problems with the Quantum Measure of Regge Calculus (and then for Dynamical Triangulations as well): many assume that the measure is

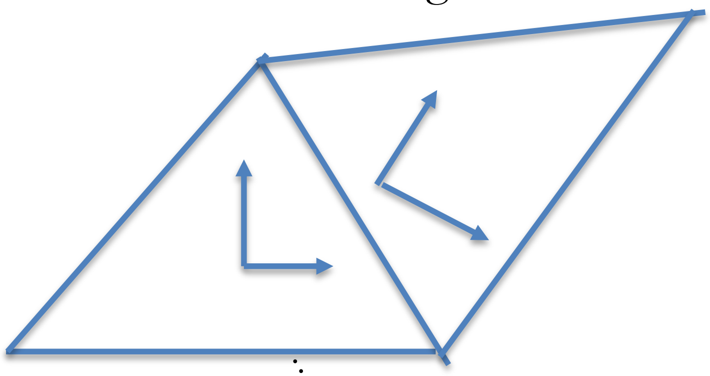
$$D[l^2] \propto l^2 dl \ (?)$$

- Regge Calculus (Dynamical triangulations) allows to implement numerical simulation of Quantum G.R. and study the phases of the Planck Universe.

- With a *disputed quantum* measure above…are these simulations correct?

QUANTUM MEASURE IN REGGE CALCULUS

- The idea consists in introducing a reference frame in each triangle (simplex) so that each vertex has a coordinate in each triangle.



- The flat metric, in each triangle, could be written as function of the lengts of the edges and the coordinates of each vertex

$$g_{\mu\nu} \equiv g_{\mu\nu}(x^\alpha, l_{ij}^2)$$

QUANTUM MEASURE IN REGGE CALCULUS

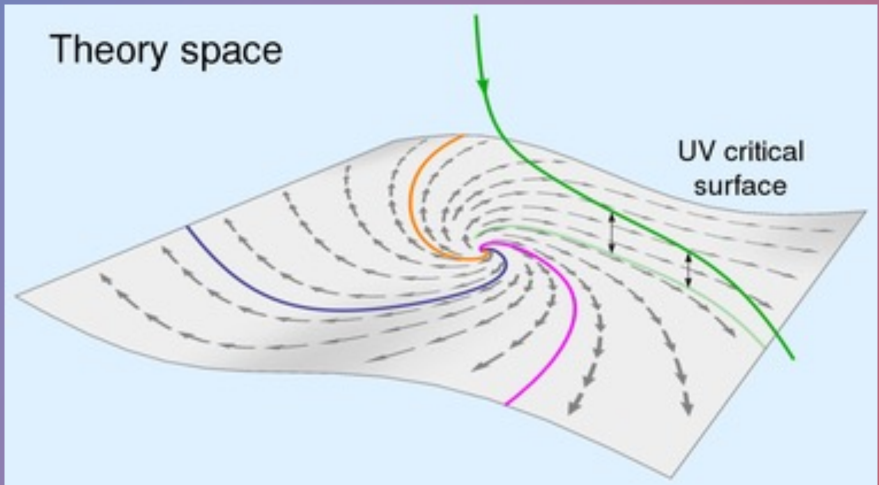
- Therefore the right measure should be

$$Z = \int D[x^\alpha] D[l_{ij}^2] e^{-S_R(l_{ij}^2, x^\alpha)}$$

- Gauge-Fixing the coordinates to the

$$Z = \int D[l_{ij}^2] J(l_{ij}^2) e^{-S_R(l_{ij}^2)}$$

- There already exists a simulations, in Dynamical Triangulations, in which they have to add a term like $J(l_{ij}^2)$, called Faddeev-Popov determinant, to make the phase transition second order



ASYMPTOTIC SAFETY APPROACH TO QUANTUM GRAVITY

- The "Asymptotic safety approach to Quantum Gravity" is based on "Weinberg conjecture" (1979). He suggested to run the coupling constant as function of a cut-off. Find a Non Gaussian Fixed Point (NGFP) in this space of parameters, define the Quantum Theory of Gravity at this point.
- d=2+ϵ: F. P. exists (Weinberg); d=4 NGFP in the Einstein-Hilbert truncation exists (Reuter and Saueressig 2002).
- "Nonperturbative renormalizability". There exist fundamental theories which are not perturbatively renormalaizable (along the line of K. Wilson's general principles of renormalization)

ASYMPTOTIC SAFETY APPROACH TO QUANTUM GRAVITY

- One needs to consider actions in Einstein General Relativity like

$$S = \frac{1}{16\pi G(k)} \int dx^4 \sqrt{-g}\,(R - 2\Lambda(k))$$

- Run the cut off k infinity. There should exist a non-Gaussian fixed point in the space of the parameters.
- Define the Quantum Theory at the non-Gaussian fixed point

VATICAN OBSERVATORY LECTURES ON QUANTUM GRAVITY 2026
OPEN QUESTIONS IN QUANTUM GRAVITY
JUNE 22-26, 2026 - SPECOLA VATICANA

Lecturers:
Sergio Cacciatori (Università dell'Insubria)
Claus Kiefer (University of Cologne)
Pierpaolo Mastrolia (Università di Padova)
Roberto Percacci (SISSA)

The quantization of space and time as GR dynamical variables is an unresolved problem. Therefore known quantization methods generate technical problems like perturbative non-renormalizability of the theory.
The lectures will explore fundamental approaches, including: canonical quantum gravity, the problem of time, amplitudes and effective field theory in quantum gravity, covariant quantum gravity. The goal is to confront and contrast these methods, fostering the exchange of ideas necessary to resolve outstanding issues in the field.

The interdisciplinary program consists of FOUR courses for a total of 20 hours and it is oriented to PhD students, postdocs and early career researchers, both in mathematics and in physics. Interaction with and among professors will also be encouraged.

Scientific & Organizing board:
Gabriele Gionti - Dean (Vatican Observatory)
Matteo Galaverni (Vatican Observatory)

Registration:
*All applications have to be submitted through the Registration Form Indico system by **15 April 2026** here: https://indico.global/event/16082*
Selected participants will be invited to pay the registration fee into the bank account that will be indicated to them.

Registration includes:
teaching material in PDF format,
lunches, coffee breaks, Guided tour and Social Dinner
EARLY FEE: € 100,00 due payment by May 30th, 2026
REGULAR FEE: € 200,00 due payment by June 15th, 2026
Participants have to find their accommodation in Albano Laziale and cover their travel expenses.

Fellowships:
Scholarships for accommodation (shared room) are offered for up to a maximum of 10 participants

Email:
specolaevents@protonmail.com

CONCLUSIONS

- A well established theory of Quantum Gravity does not exist yet. Lack of experimental tests

We have analyzed several approaches to Quantum Gravity

1. Canonical Quantum Gravity
2. Loop Quantum Gravity (Mathematical problems)
3. String Theory (no supersymmetry found..)
4. Quantum Regge Calculus
5. Asymptotic Safety (quite young…evidence for the existence of the Non-Gaussian Fixed Point, no final proofs).

- More work appears needed to define a final theory of Quantum Gravity. This theory should reproduce all semi-classical results (e.g. Black Hole entropy) and it should have classical Einstein General Relativity as a limit when $\hbar \longrightarrow 0$.