Modified Newtonian Dynamics (MOND)
- a competitor to the Dark Matter paradigm

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Teramo June 2026

$$c \times H_0$$

Hawking 1972: event horizons of black holes have temperature

Bekenstein 1972: event horizons of black holes have entropy

Unruh 1976: an acceleration heats up space-time

Gibbon & Hawking 1977: cosmological event horizons have temperature and gravity

Jacobson 1996: Einstein equation as thermodynamic equation of State

Alonso-Serrano+ 2025: space-time thermodynamics

$c \times H_0$ - surface gravity of the event horizon of a de Sitter universe

Content

- Introduction: dark matter in cosmology

- MOND: the beginnings

- the Baryonic Tully-Fisher Relation
  for spiral galaxies

- MOND and early-type galaxies

- Gravitational lensing

- theoretical reasoning: no consent

Introduction: The "Standard Cosmology"

Standard cosmology results as a fit of
objects with known distances to the "Friedmann equations"

- **homogeneous metric**: same density everywhere
- isotropic

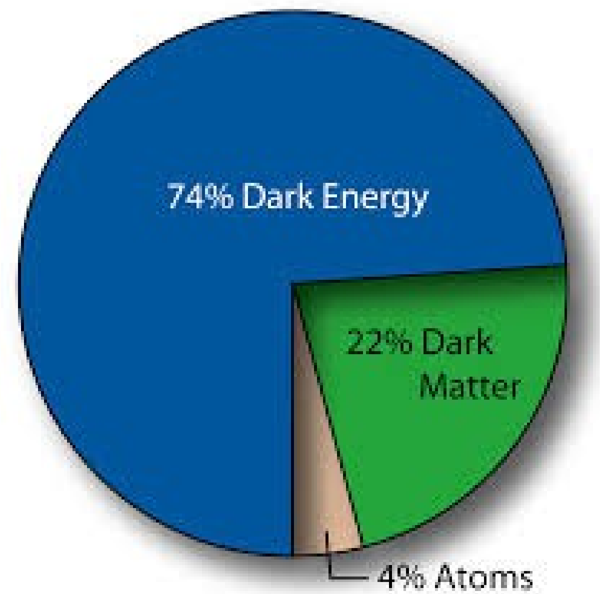
Data (Cosmic Microwave Background
+ Supernovae Ia)

+

$$H^2 = H_0^2\left(\Omega_{0,M}\, a^{-3} + \Omega_{0,k}\, a^{-2} + \Omega_{0,\Lambda}\right)$$

=

$$H_0, \Omega_{0,M}, \Omega_{0,\Lambda}$$



for a flat Universe today

$$1 = \Omega_{0,M} + \Omega_{0,\Lambda}$$

Conclusion

The Universe is (approximately) a mystery

Possible part of the solution

- Dark Energy is an artefact from describing an inhomogeneous Universe by Friedmann models
- Friedmann models do not respect structure formation

  Buchert 2018 - https://arxiv.org/pdf/1810.09188

- cosmological particle Dark Matter does not exist either
  - there is no evidence for its existence

Dark Matter

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Abstract

We review observational, experimental and theoretical results related to Dark Matter.

524 pages

Challenging points for particle Dark Matter
- weakly interacting, stable, massive(?)

--- the standard model does not know such particle

--- they have not been found in spite of very ambitious searches

--- no Gamma-ray detection from hypothetical decay products
HAWC: Galactic centre: Alvaro+ 2025, eprint arXiv:2509.06210
Dwarf galaxies: Abdollahi+ 2026, JCAP 3,35

--- they must explain the extremely close relation of baryonic and Dark Matter in galaxies (Baryonic Tully-Fisher relation)

--- they must generate gravity

The main protagonists

Theory: Moti Milgrom, Jacob Bekenstein, Bob Sanders, C. Skordis, Pavel Kroupa..

Observers: Vera Rubin, Stacy McGaugh, Erwin de Blok, Federico Lelli...

Reviewers:

P. Kroupa 2015. Galaxies as simple dynamical systems: observational data disfavor dark matter and stochastic star formation, Canadian Journal of Physics 93, 169

I. Banik and HS. Zhao, 2022, From galactic bars to the Hubble tension – weighing up the astrophysical evidence for Milgromian gravity. Symmetry 14, 1331

B. Famaey and S.S. McGaugh, 2012, Modified Newtonian Dynamics (MOND): Observational Phenomenology and Relativistic Extensions. Living Rev. Relativ., 15, 10

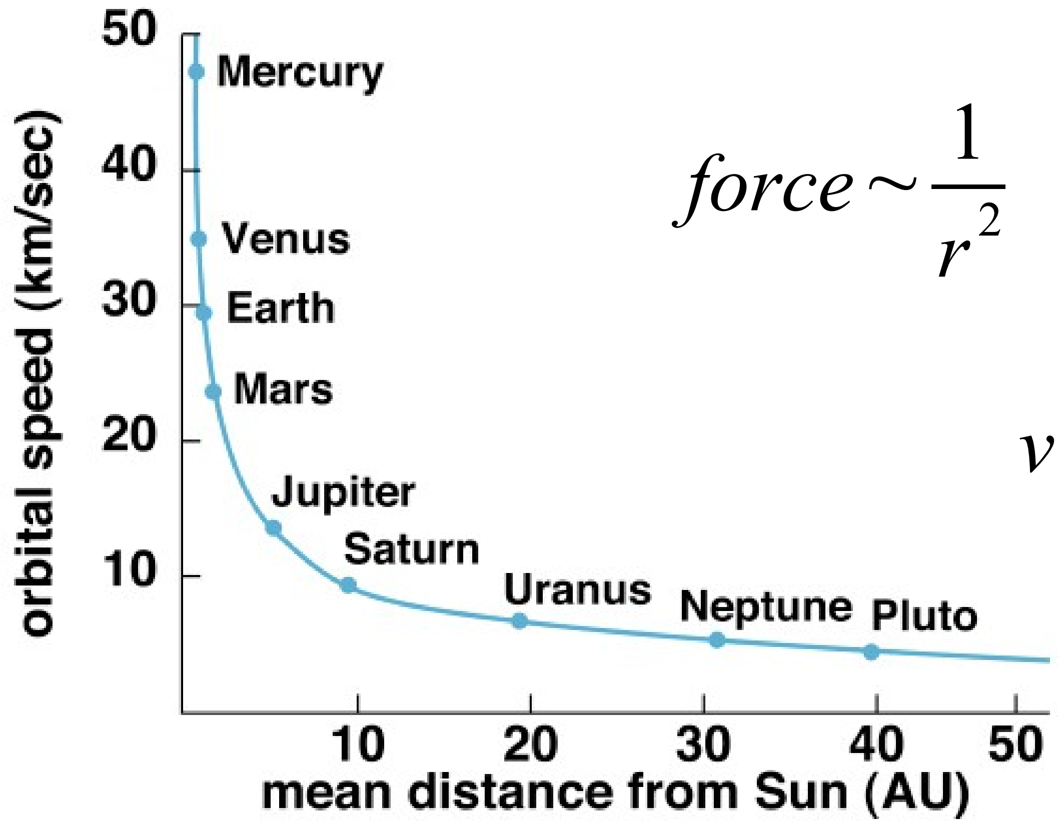
Kroupa+2023. The many tensions with dark-matter based models and implications on the nature of the Universe, 2023.eppg.confE.231K

Richtler+2024, The isolated elliptical galaxy NGC 5812 – MOND or dark matter? AN 34530081

MOND- the white paper : Federico Lelli +55 co-authors, 2026

240 pages of text (50 pages theory) 1048 references

Rotation curve of the Solar System



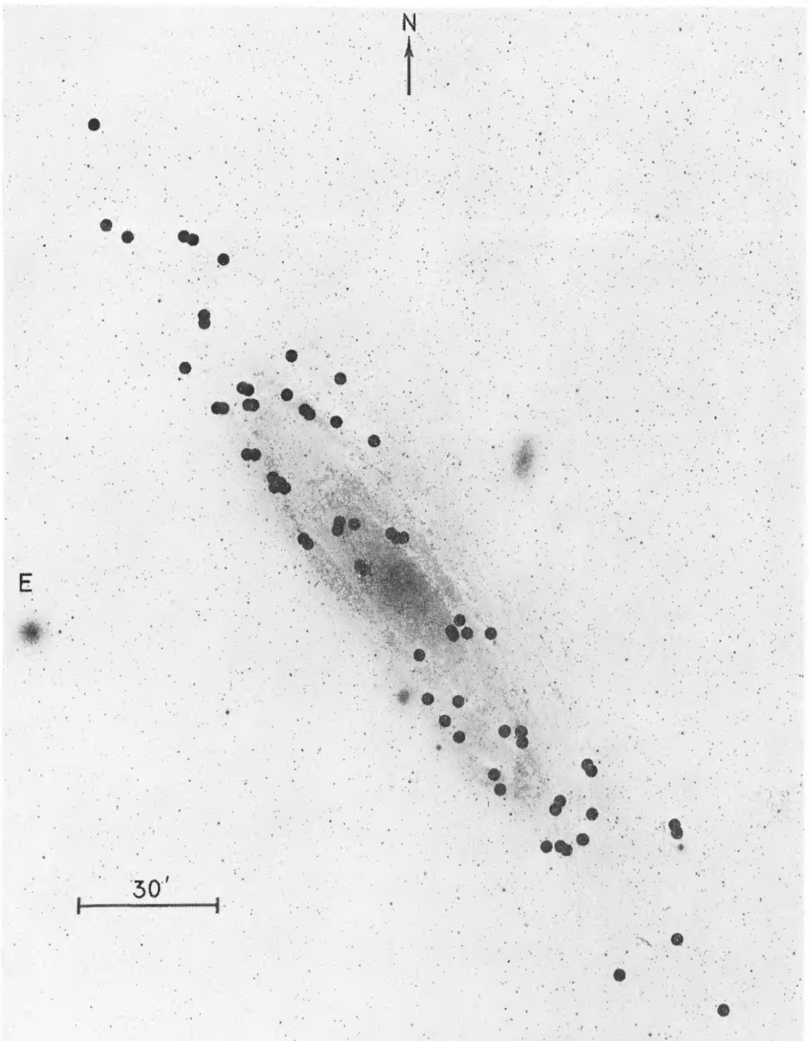
(b)

$$force \sim \frac{1}{r^2}$$

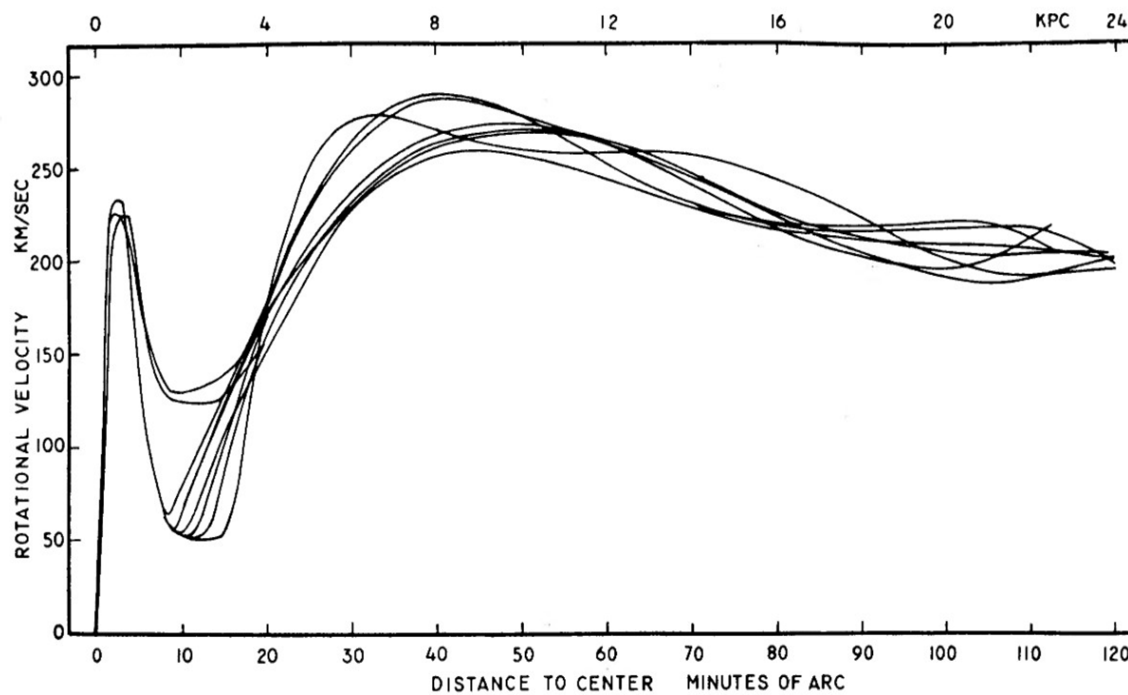
$$v_{circular} = \sqrt{\frac{G\,M_{sol}}{r}}$$

No mass discrepancy - no dark matter

Rubin&Ford 1970 (ApJ 159,379): ROTATION OF THE ANDROMEDA NEBULA FROM A SPECTROSCOPIC SURVEY OF EMISSION REGIONS*




Vera Rubin ca 1992, photograph by Mark Godfrey, credit: Carnegie Institution, Department of Terrestrial Magnetism.



No mention of mass discrepancies

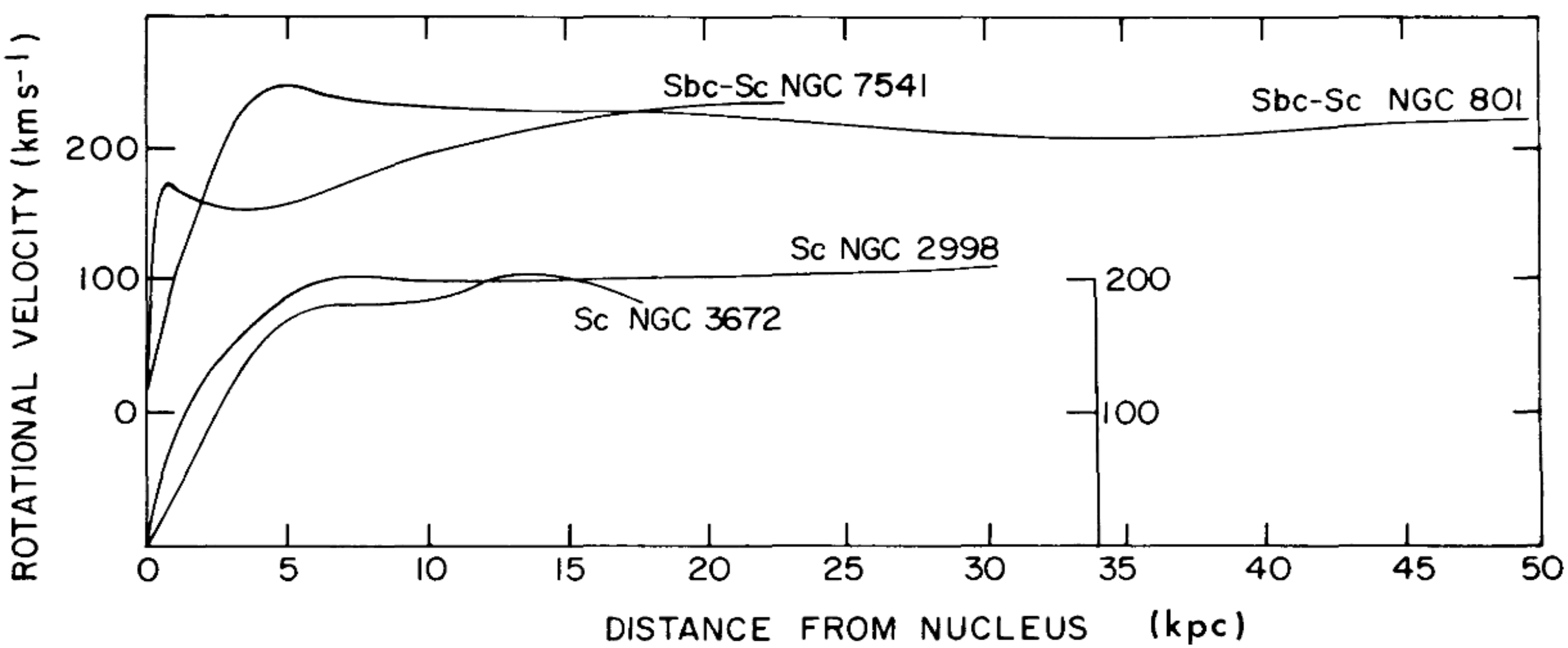
The beginnings

What was known in 1983?

– M/L of galaxies higher than expected from populations
  Faber&Gallagher (1979, ARAA 17,135)

"After reviewing all the evidence, it is our opinion that the case for invisible mass in the Universe is very strong and getting stronger"

Rubin+1978: Extended rotation curve of high luminosity spiral galaxies

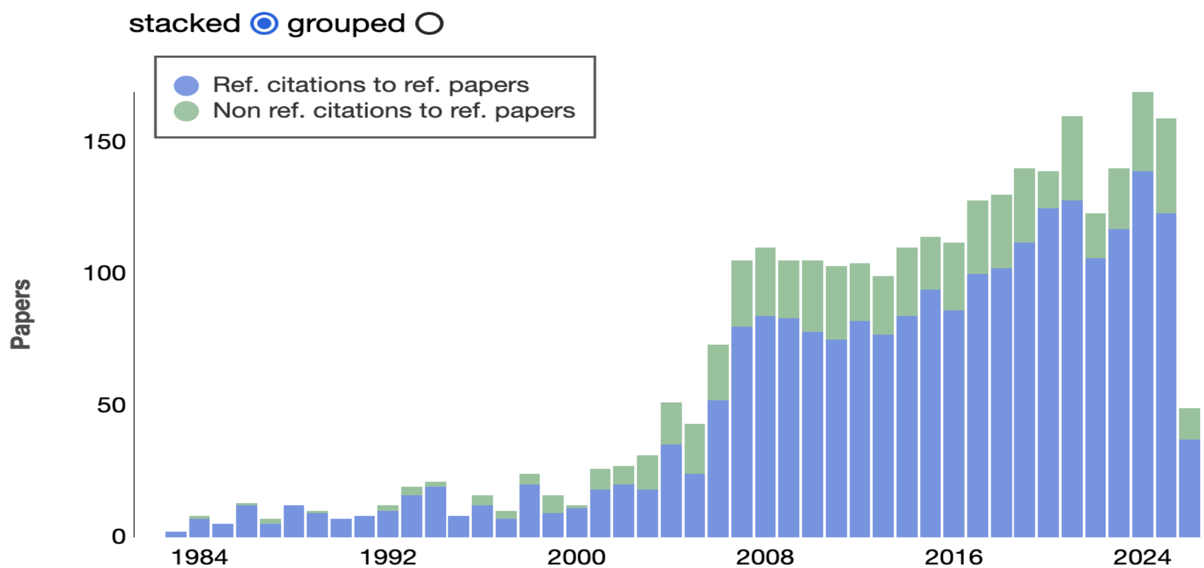


Milgrom 1983: A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis ApJ 270, 365 (2892 citations)

Milgrom 1983: A modification of Newtonian dynamics - Implications for galaxies ApJ 270, 371 (1018 citations)

Milgrom 1983: A modification of Newtonian dynamics - Implications for galaxy systems ApJ 270, 384 (557 citations)

MOND-pages by Stacy McGaugh
https://astroweb.case.edu/ssm/mond/



Teramo June 2026

Modified Newtonian Dynamics - Basics

Milgrom's recipe (1983) for an asymptotically flat rotation curve: replace the Newtonian acceleration by

$$g_{MOND} = \sqrt{g_{New} \cdot a_0} \quad \text{if} \quad g_{MOND} \ll a_0$$ "deep MOND regime"

a_0: constant of nature with dimension cm/s^2

empirically $$\boxed{a_0 \approx 1.3 \times 10^{-8} cm/s^2 \approx c \times H_0}$$

for circular velocities: $$v_{c,MOND}^2 = g_{MOND} R = \sqrt{G M(R) a_0}$$

---> asymptotically constant circular velocity

---> baryonic Tully-Fisher relation $$M_{baryon} = \frac{1}{G a_0} v_{c,MOND}^4$$

A quadratic Lagrangian (AQUAL)

Problem: conservation laws violated with simple replacement

Solution: construction of a Lagrangian, from which equations of motion can be derived

$$\vec{\nabla} \cdot \left(\frac{|\vec{\nabla}\Phi|}{a_0} \vec{\nabla}\Phi \right) = 4\pi G_N \rho.$$

Bekenstein and Milgrom 1984 (ApJ 286,7): DOES THE MISSING MASS PROBLEM SIGNAL THE BREAKDOWN OF NEWTONIAN GRAVITY?

In configurations of high symmetry (spherical mass distribution, disk)

$$g_M(r) = \mu\left(g_N(r)/a_0\right) g_N(r)$$

Where Å(x) is an arbitrary function Å(x) = 1 for $g_M >> a_0$

Å(x) = x for $g_M << a_0$

How to test MOND?

Galaxies with disks (spiral galaxies) – rotation curves

Galaxies without disks (elliptical galaxies) – virial theorem
Integrated galaxy light and "dynamical tracers"

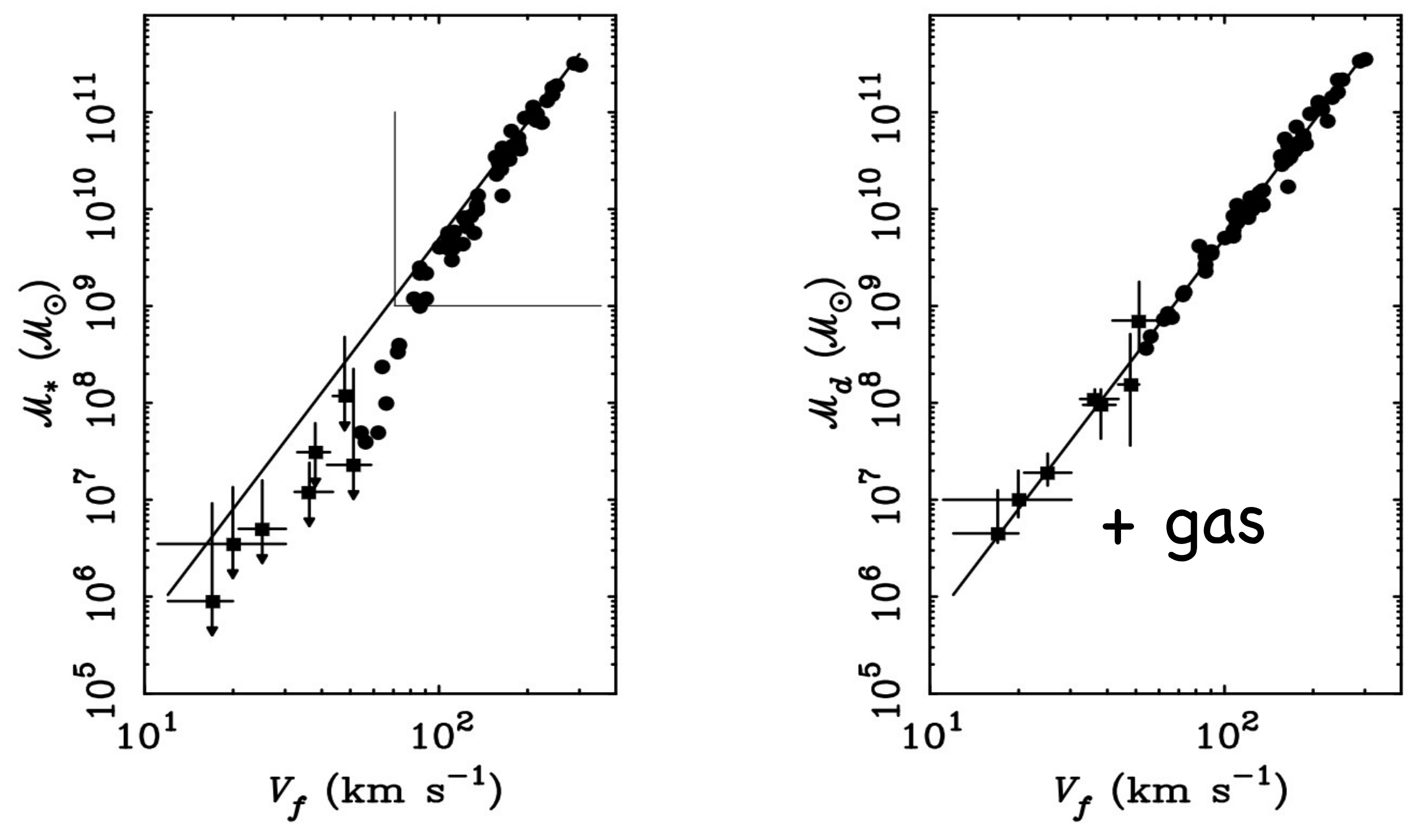
Bright galaxies/galaxy clusters with hot gas -
Assumption of hydrostatic equilibrium

Both isolated galaxies and galaxy clusters -
Gravitational lenses

Stacy McGaugh: Baryonic Tully-Fisher relation for galaxies with extended HI-rotation curves 2005, ApJ 632, 859

"Baryonic" masses (stars + gas) vs. rotation velocities in the flat regime

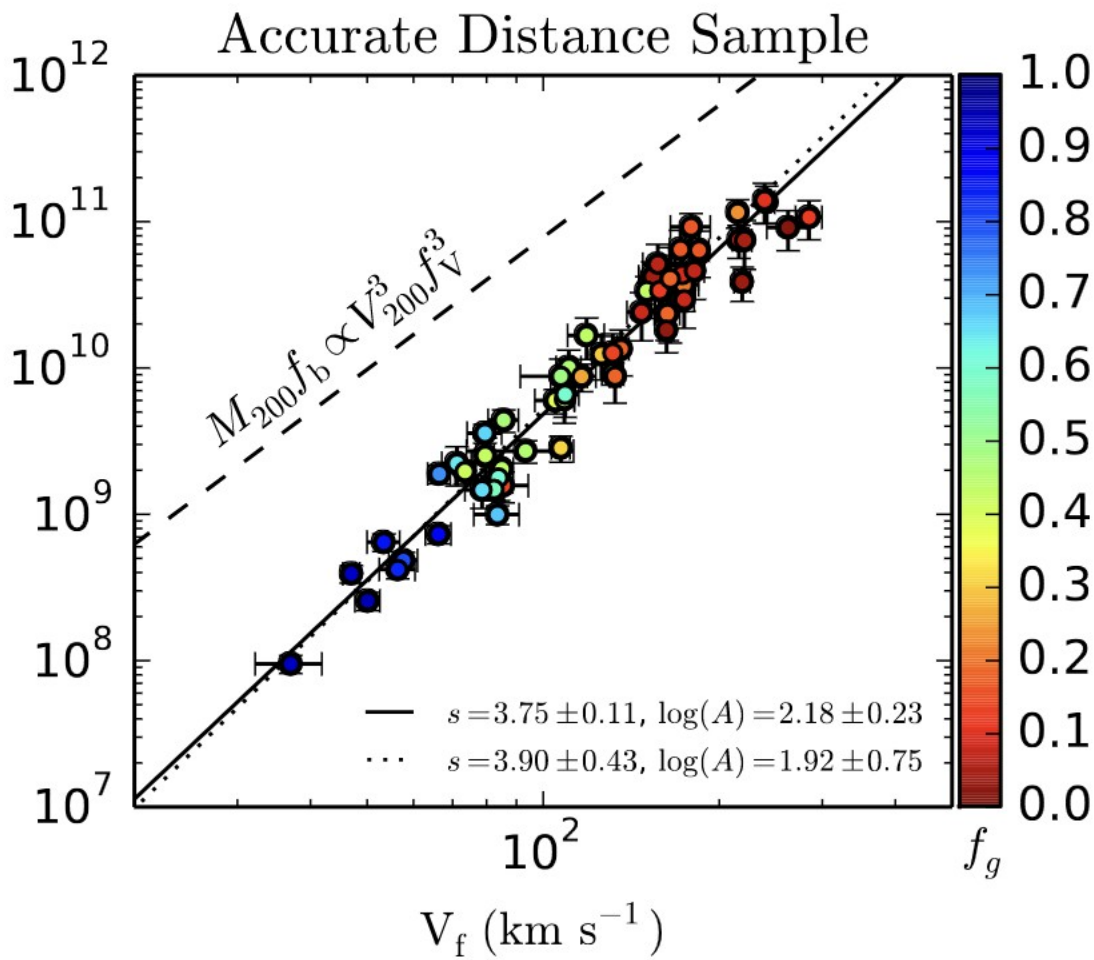
Stellar masses BARYONIC TULLY-FISHER RELATION Dynamical masses



$$M_{baryon} = 50 \times v_f^4$$

Lelli+2016   BTFR from 125 disk galaxies with high data quality
Color coding: gas mass fraction

LCDM



The masses of elliptical galaxies

M87

No disk assumption of spherical symmetry is necessary

Observational parameters:

Integrated galaxy light

 projected velocity

 projected velocity dispersion

Dynamical tracers: radial velocities of globular clusters
planetary nebulae

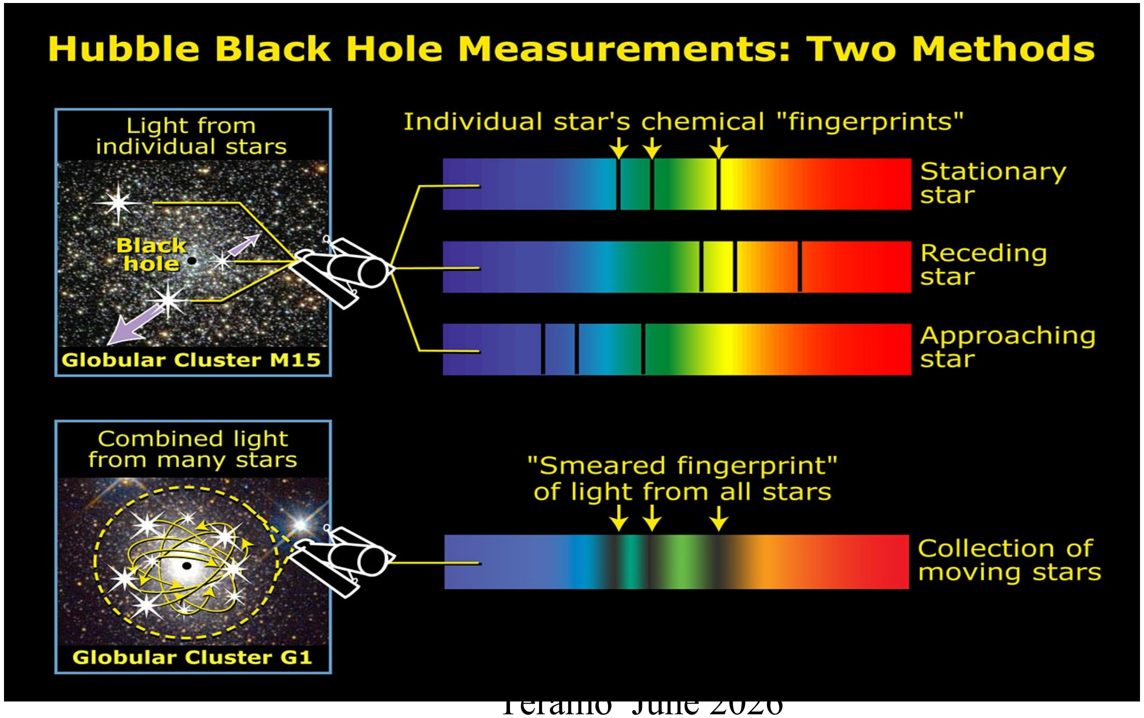
Projected velocity dispersion

Gravitating system in virial equilibrium

- velocities follow a Maxwellian distribution
- along the line-of-sight = 1-dim = radial velocities: Gaussian

Elliptical galaxies are fully transparent

- spectral lines are broadened by the Gaussian



Jeans models I

Fundamental – collisionless Boltzmann equation

$$\frac{d\,f}{dt} = 0$$

expansion into velocity moments and in spherical coordinates without rotation

Density of tracer population

 spherical Jeans-equation

$$v^2_{circular} = \frac{M(r)\,G}{r} = -\sigma_r^2 \left[\frac{d\ln\rho}{d\ln r} + \frac{d\ln\sigma_r^2}{d\ln r} - 2\beta \right]$$

Anisotropy parameter $\beta = 0$ isotropy

$\beta = -\infty$ tangential anisotropy

$\beta = +1$ radial anisotropy

Jeans models II

modelling M(r) in the spherical, non-rotating case

solution of the spherical Jeans-equation

anisotropy

$$\ell(r)\,\sigma_r^2(r) = \mathrm{G}\int_r^{\infty} \ell(s)\,M(s)\frac{1}{s^2}\left(\frac{s}{r}\right)^{2\beta}\mathrm{d}s$$

profile of the tracer population

measurements: projected velocity dispersion

projected tracer population profile

projection integral

$$\sigma_{\mathrm{los}}^2(R) = \frac{2}{N(R)}\left[\int_R^{\infty}\frac{\ell\sigma_r^2\,r\,\mathrm{d}r}{\sqrt{r^2-R^2}} - R^2\int_R^{\infty}\frac{\beta\ell\sigma_r^2\,\mathrm{d}r}{r\sqrt{r^2-R^2}}\right]$$

Line-of-sight velocity dispersion

Galaxy Sample

isolated ellipticals

NGC 6411 Richtler+2024 MONDian
NGC 7796 Richtler+2015 MONDian
NGC 7507 Lane+2015 not MONDian!
NGC 5812 Richtler+2024 can be MONDian
NGC 2865, NGC 1132... ?? in prep.

Group galaxies

NGC 4636 Schuberth+2012 MONDian > 400 radial velocities

central galaxies

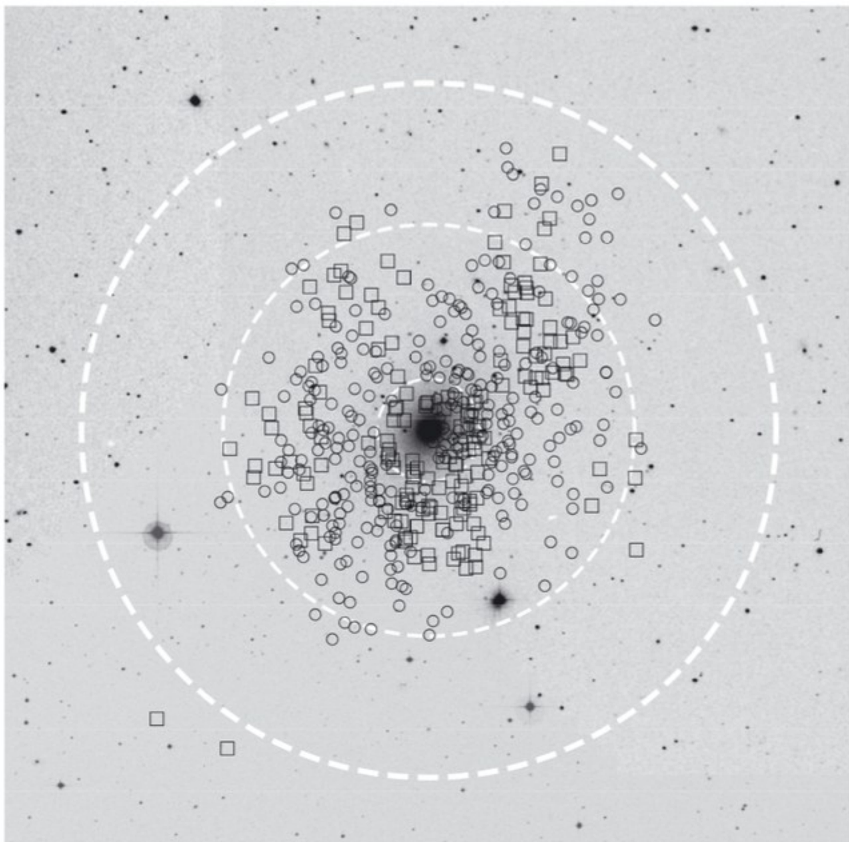
NGC 1399 Fornax Schuberth+2010
NGC 3311 Hydra I Richtler+2011, 2020

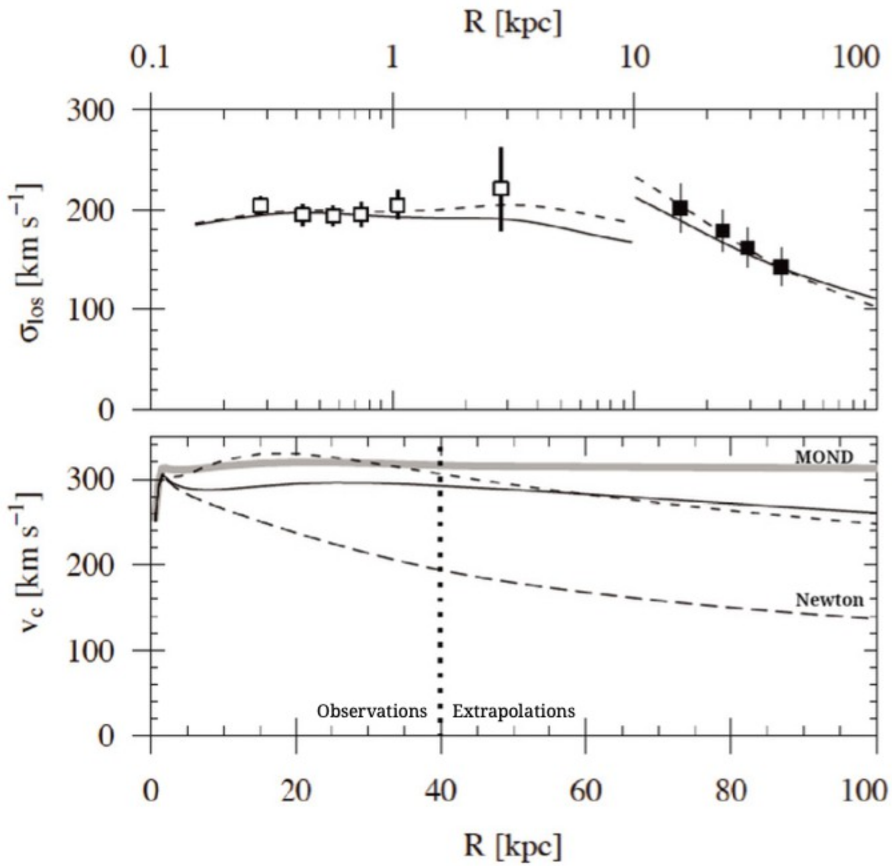
Sources of uncertainty

- measurement of velocity dispersions not trivial
- spherical symmetry ??
- rotation
- distance
- ideal Jeans world?

NGC 4636 – Schuberth 2012 A&A 544,115

460 globular cluster velocities - the largest sample for a non-central galaxy



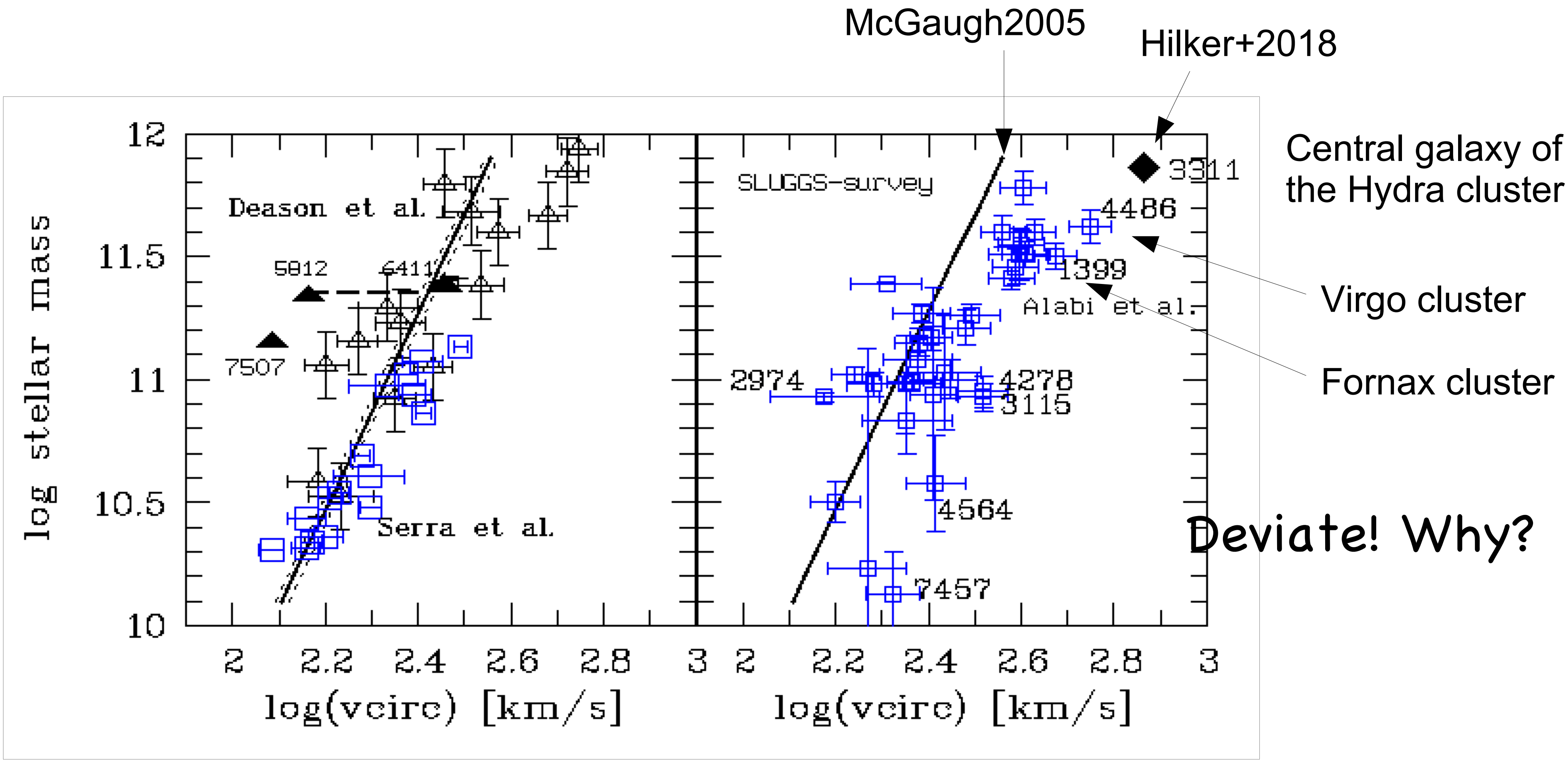


MONDian!

The Baryonic Tully-Fisher Relation
- early-type galaxies Richtler+2024

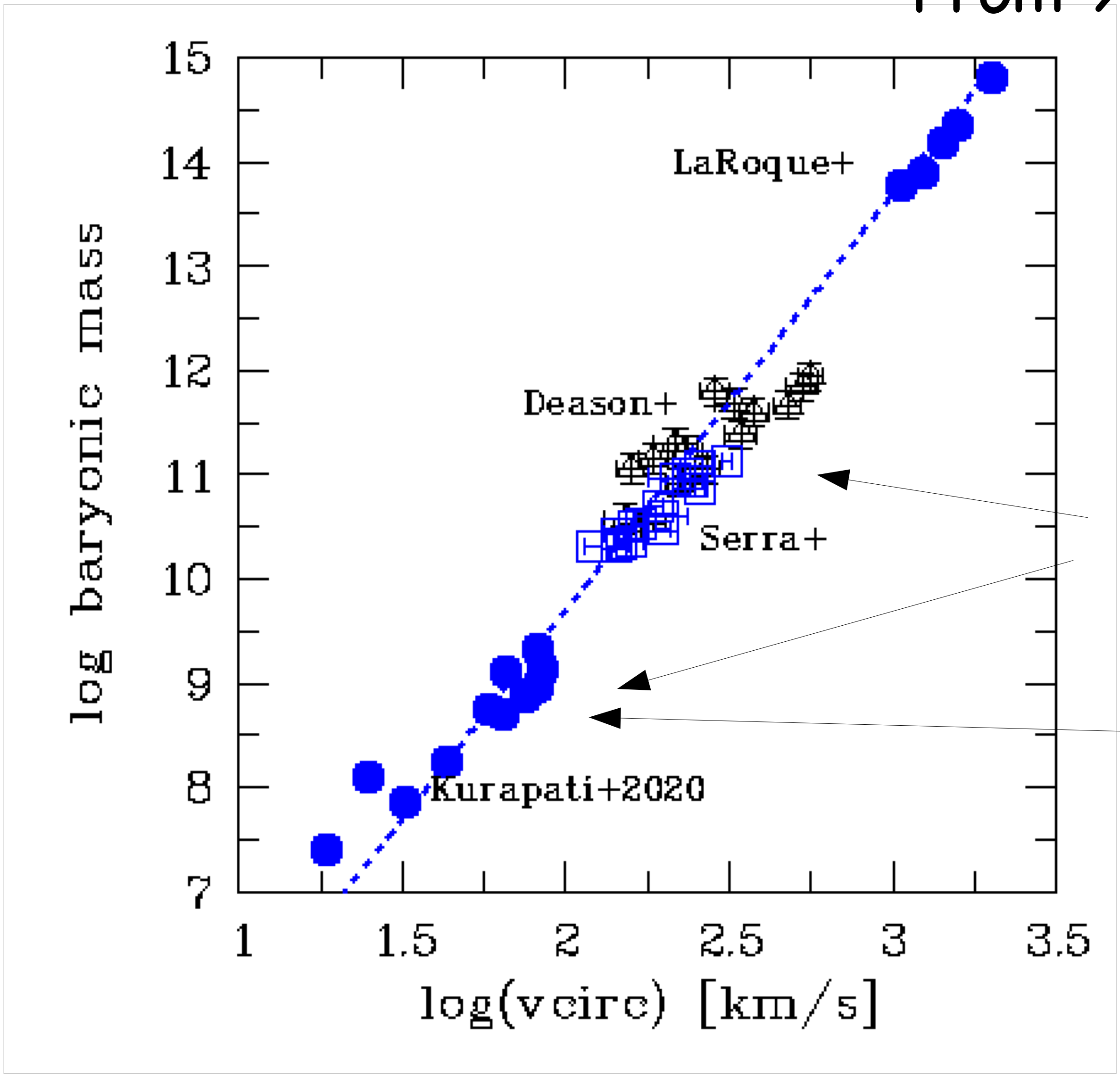
Samples of early-type galaxies with masses: Deason+2012
Serra+2016 (with HI disks)

SLUGGS survey: globular cluster velocities Alabi+2016



The baryonic Tully-Fisher relation with galaxy clusters from LaRoque+2006, and the void dwarf galaxy sample of Kurapati+2020

From X-rays



Galaxy clusters

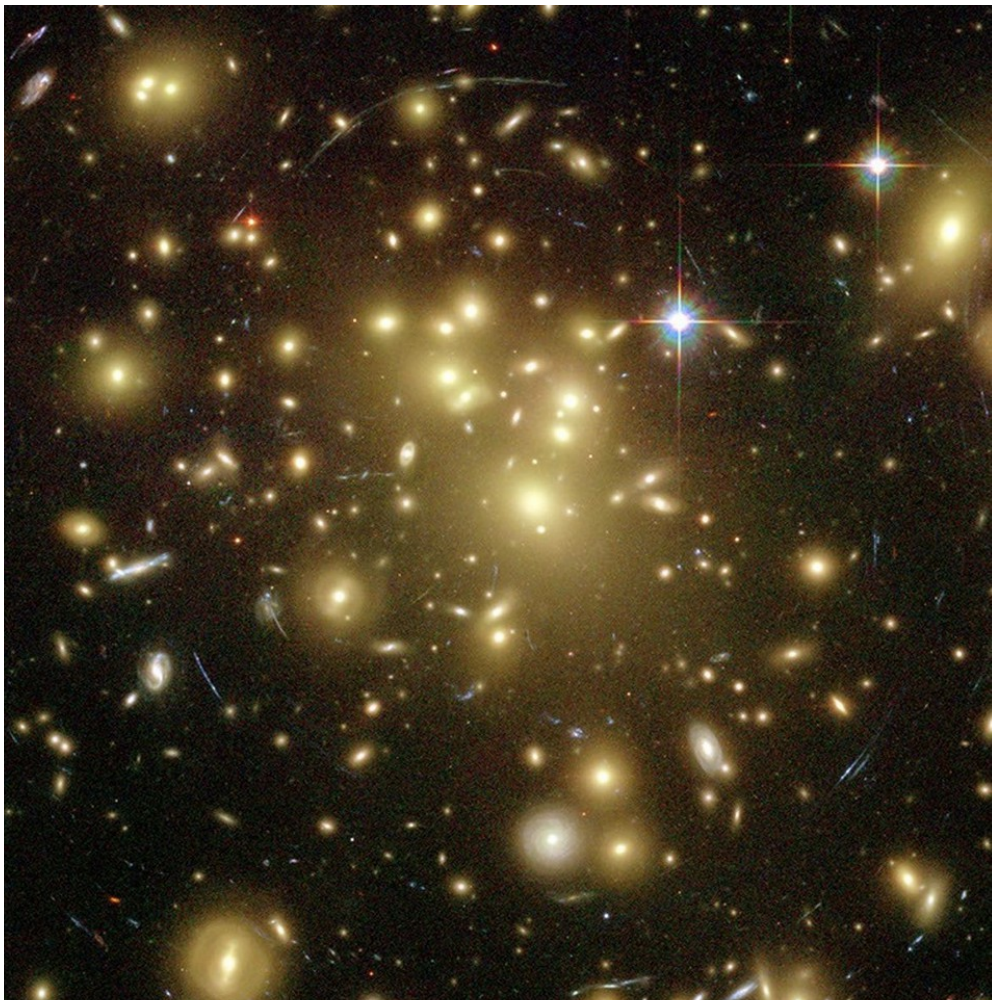
Not a fit!

at 5 effective radii

gas rich dwarf galaxies in a void!

Galaxy clusters

Within the cosmic web, galaxy clusters are found at the intersections of cosmic filaments



Abell 1689

Z = 0.16

Full of lenses

Mass about 10**15 solar masses

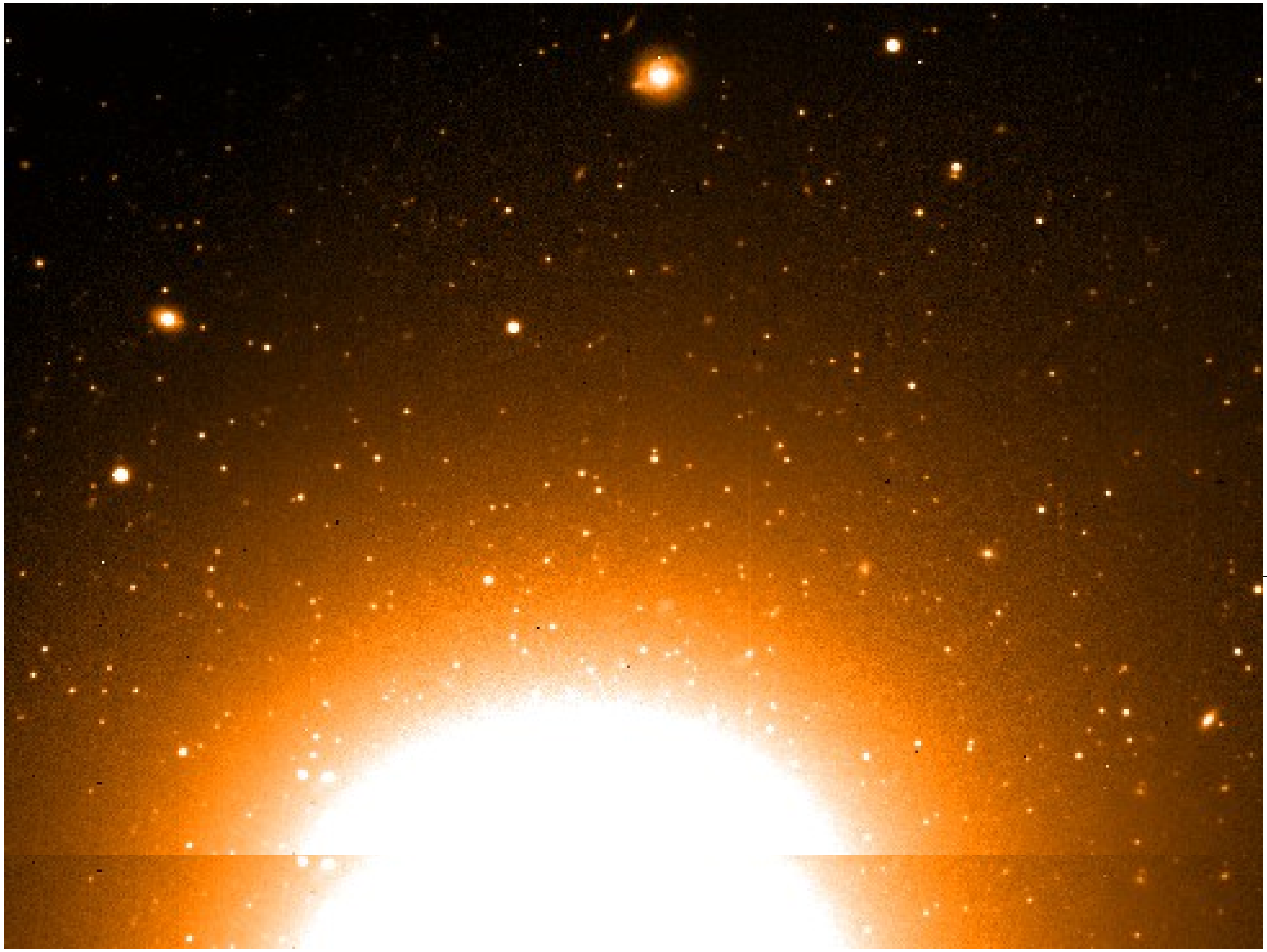
80% of the mass is gas of 10**5 K

Typical acceleration about a_0
Not MONDian!

Central galaxies have rich globular cluster systems

NGC 1399 in the Fornax cluster

Very Large Telescope (Cerro Paranal)



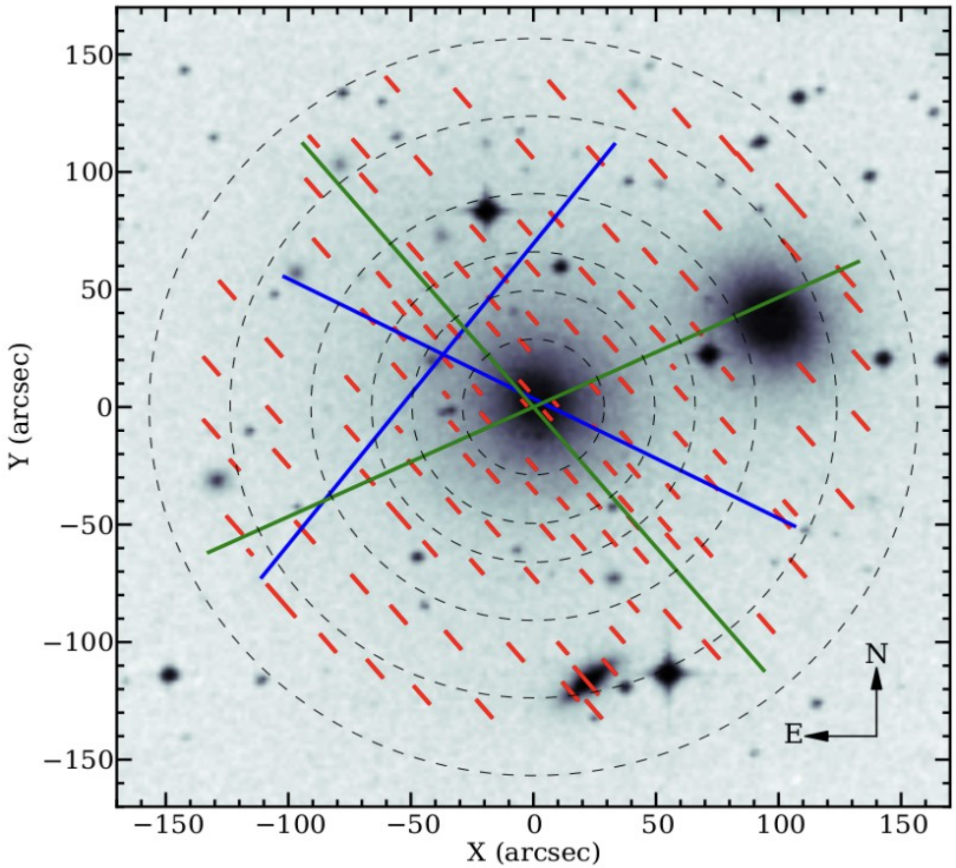
Approx. 6000 globular clusters

Schuberth et al. 2010

Inner core of the Hydra I cluster with NGC 3311 as the central galaxy

Hilker+2018, A&A 619,A70

Kinematic complexity in a rising velocity dispersion profile around the cD galaxy NGC 3311

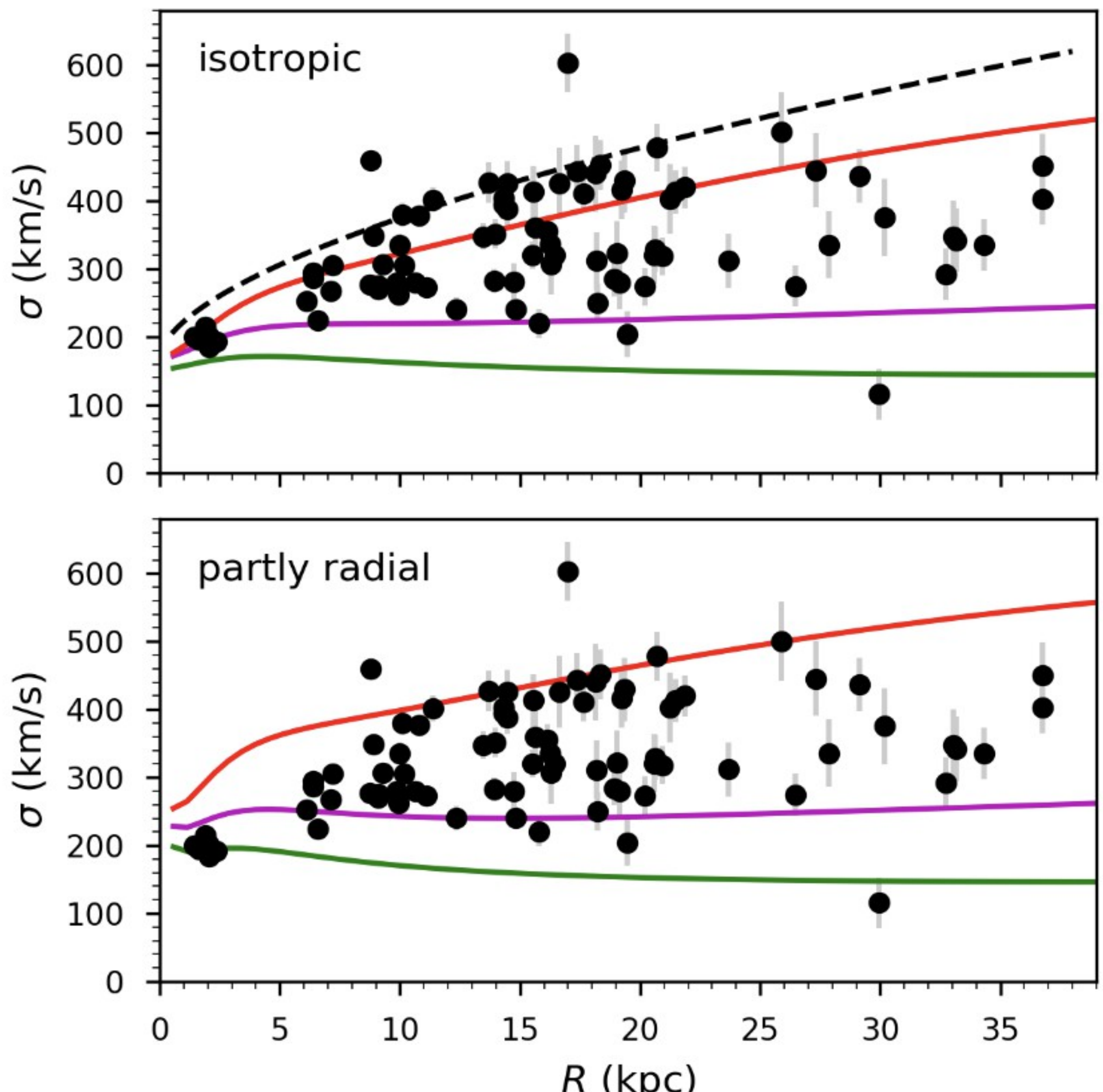


Spectroscopic slits from

FORS2
Very Large Telescope,
Cerro Paranal

Jeans models for NGC 3311 Hilker+2018

Inner region out to 40 kpc



Dark matter halo

MOND

Only stars

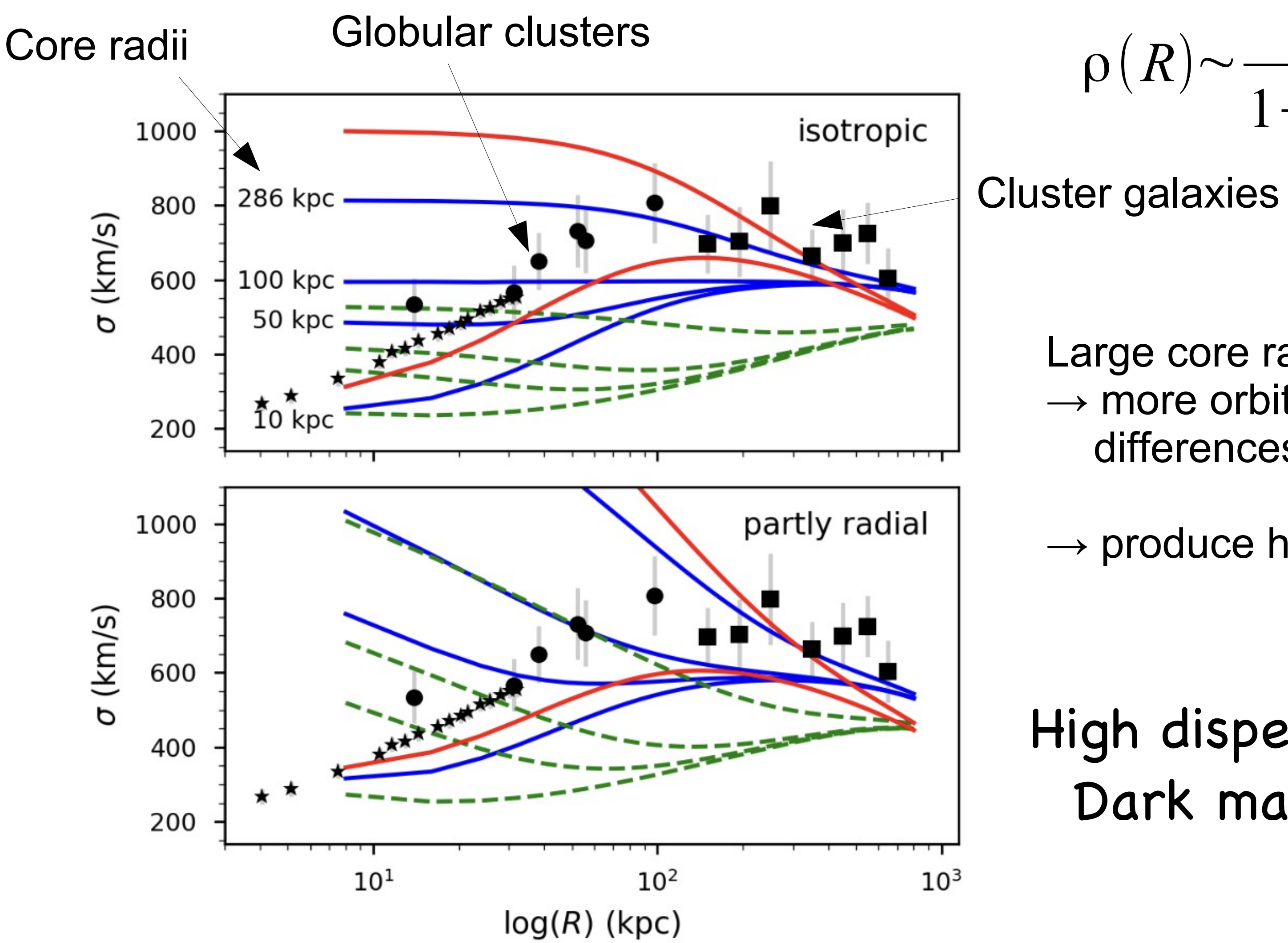
Dark matter halo

MOND

Only stars

No ideal Jeans-world

Jeans models for the entire Hydra cluster (stars and gas) for different tracer density profiles out to 1 Mpc



$$\rho(R) \sim \frac{1}{1+(R/R_{core})^2}$$

Large core radii
→ more orbits with large potential differences

→ produce high vel. dispersions

High dispersions do not mean Dark matter halos!

Gravitational lenses

Strong lensing: multiple or strongly deformed background galaxy images

- visible mostly in galaxy clusters
- no MONDIan environment

Weak lensing – statistical weakly deformed images of background galaxies

Interesting for MOND – isolated galaxies in regions of redshift surveys

Brouwer+2021, A&A 650,A113 : The weak lensing radial acceleration relation: Constraining modified gravity and cold dark matter theories with KiDS-1000

Indefinitely Flat Circular Velocities and the Baryonic Tully-Fisher Relation from Weak Lensing

(Mistele+2024, ApJL 969,L3)

Baryonic Tully-Fisher relation from weak lensing

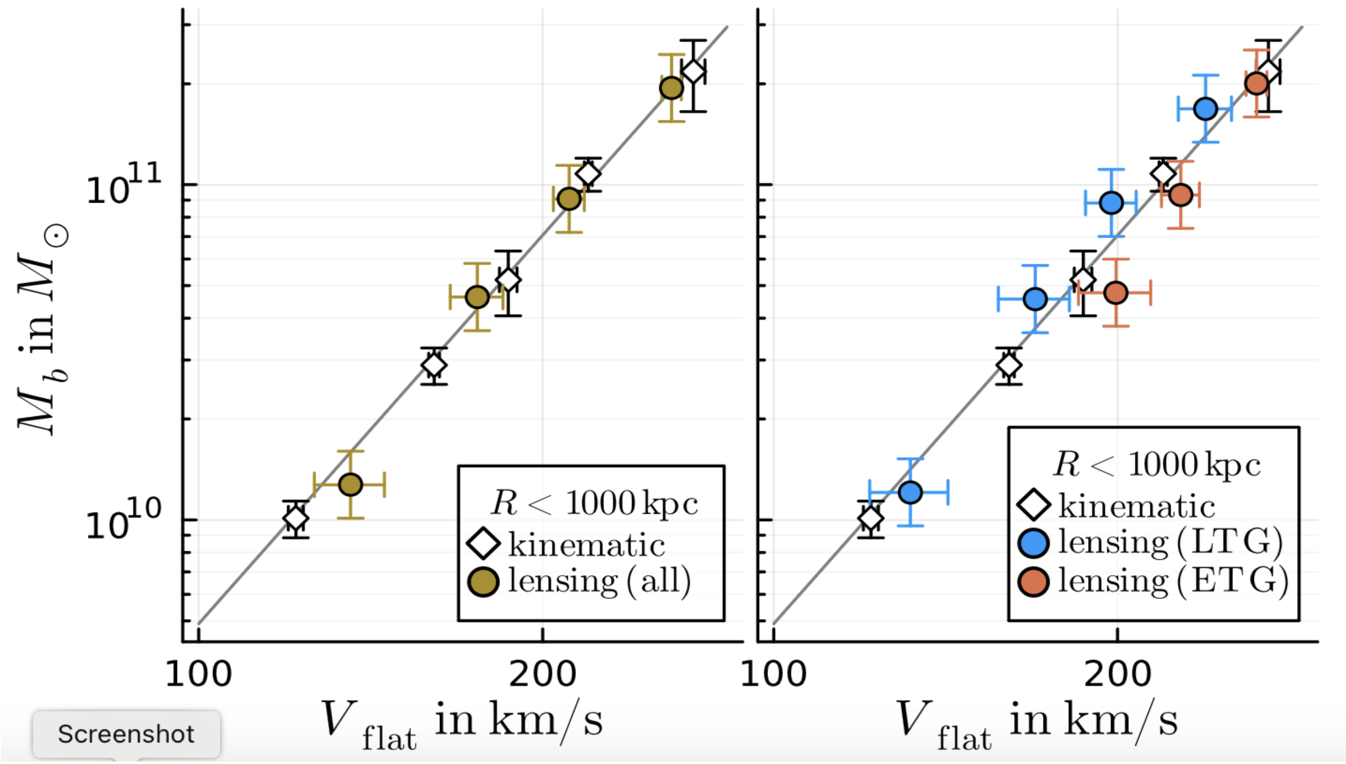
Mistele+2024

LTG: late type galaxies - galaxies with disks

Four mass bins total of 75582 lenses

ETG: Early type galaxies - elliptical galaxies

Four mass bins total of 53894 lenses

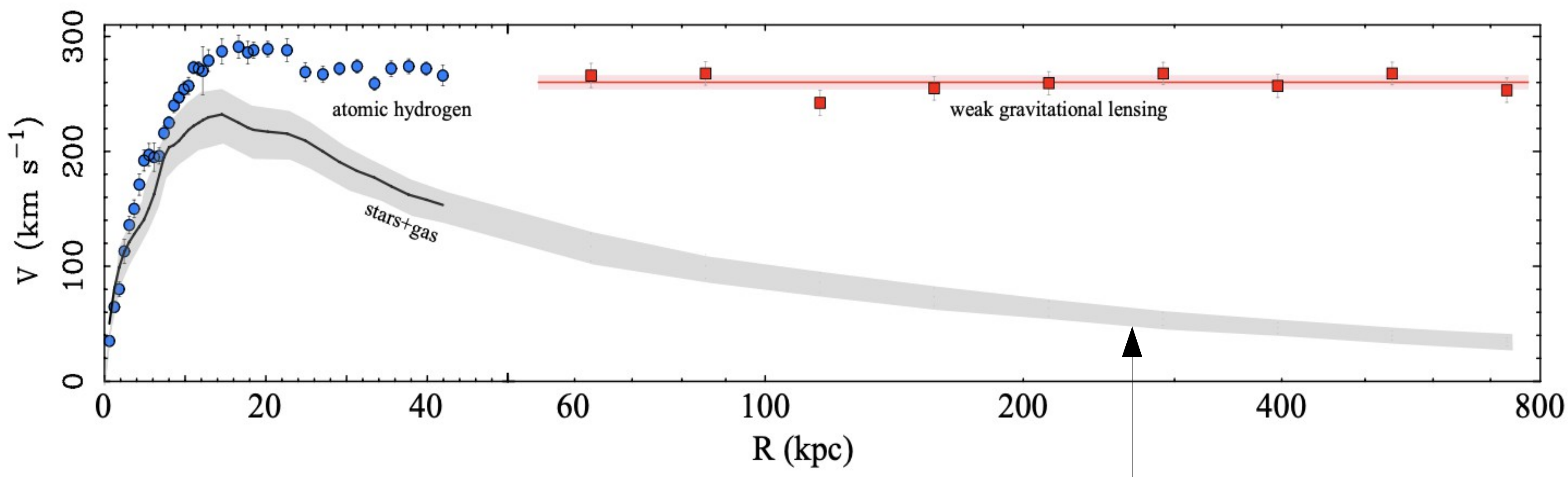


Kinematic sample from Lelli+2016

Flat rotation velocities
Out to 1 Mpc

Indefinitely Flat Circular Velocities and the Baryonic Tully-Fisher Relation from Weak Lensing (Mistele+2024, ApJL 969,L3)

Rotation curve of UGC 11455 (Mistele+ 2024)



Virial radius of a putative dark matter halo

Flat circular velocity beyond virial radius of dark matter halo

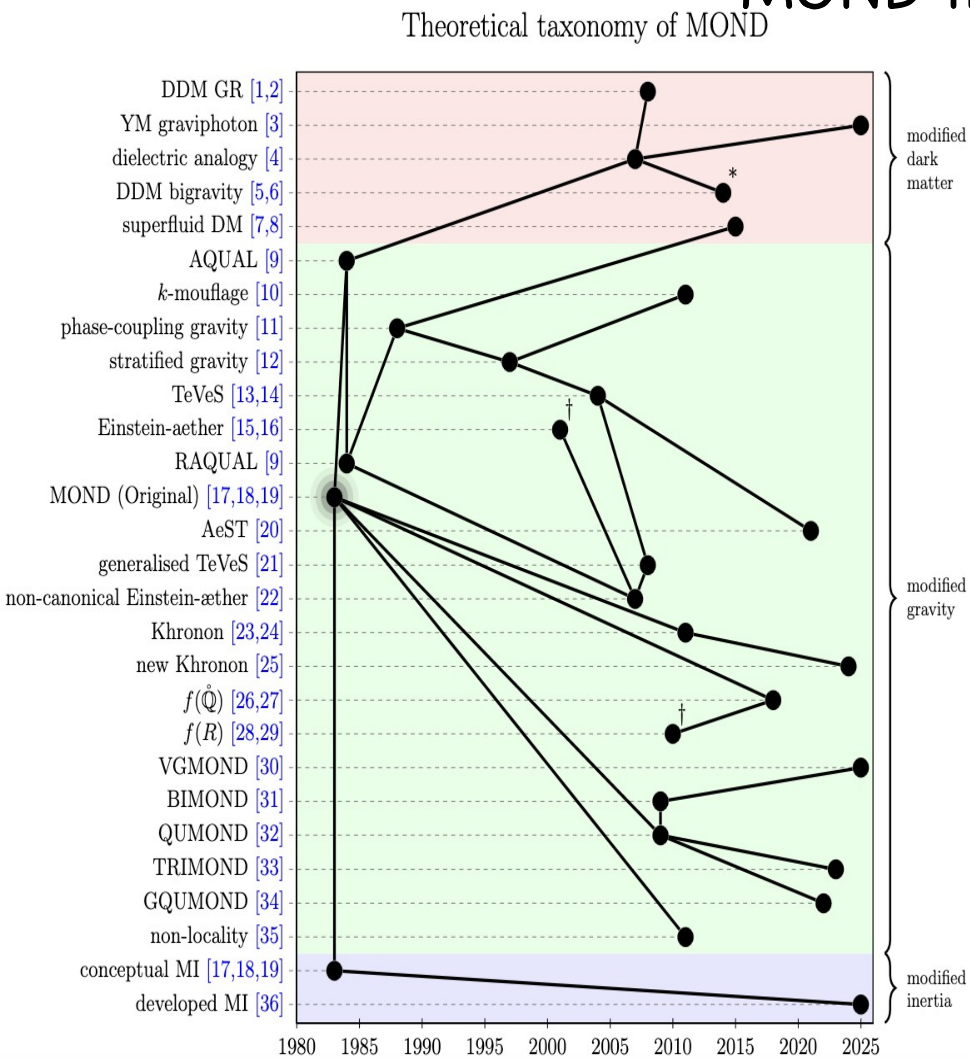
No dark matter!

Theory

MOND – white paper lists

MOND modified gravity theories 27

MOND modified inertia 1 (Milgrom)



* Can also be viewed as modified gravity.

† Motivated independently of MOND.

e 2026

gravity

F = m x a

inertia

Relativistic theories for MOND as modified gravity

Modified gravity – General Relativity (GR) breaks down in case of low accelerations

– needs additional gravitational degrees-of-freedom beyond the spin-2 metric of GR

- adding vector and scalar fields

TeVeS (Bekenstein 2004) – first relativistic MOND theory

- GW 170817 : neutron star merger
- gravitational waves couple to the same metric as photons
- TeVeS ruled out, but marked the construction of more satisfactory theories

MOND as modified inertia from quantum vacuum

Hawking 1972: Schwarzschild horizons radiate
Bekenstein 1973: Black holes and entropy (6357 citations)

Gibbons&Hawking 1976: Any causal horizon in space-time has a temperature and surface gravity

Fulling(1973), Davies (1974), Unruh (1976):
An accelerated observer receives a thermal radiation with temperature

$$T = \frac{\hbar a}{2\pi c k_B}$$

De Sitter universe has an event horizon with surface gravity

$$a_\Lambda = c^2 \sqrt{\Lambda/3} \approx 6.1 \times 10^{-10} m/s^2$$

A accelerated observer in a De Sitter universe (matter density zero, only cosmological constant) sees a temperature

$$T_\Lambda = \frac{\hbar}{2\pi c k}\sqrt{(a^2 + a_\Lambda^2)}$$

Narnhofer, Peter, Thirring 1996

Milgrom (1999) proposes:

- quantum vacuum defines an absolute inertial frame
- the force in Newton's law is proportional to the excess temperature with respect to a de Sitter background

$$\begin{aligned} F &= m a \\ &= m 2\pi c \frac{k_B}{\hbar}[T_\Lambda(a) - T_\Lambda] \\ &= m\left(\sqrt{(a^2 + a_\Lambda^2)} - a_\Lambda\right) \\ &= m a \mu(a/a_0) \end{aligned}$$

MOND

MONDian modification of inertia

$$a_0 = 2\,a_\Lambda$$

$$\mu(x) = \frac{\sqrt{\left(1+4x^2\right)}-1}{2x}$$

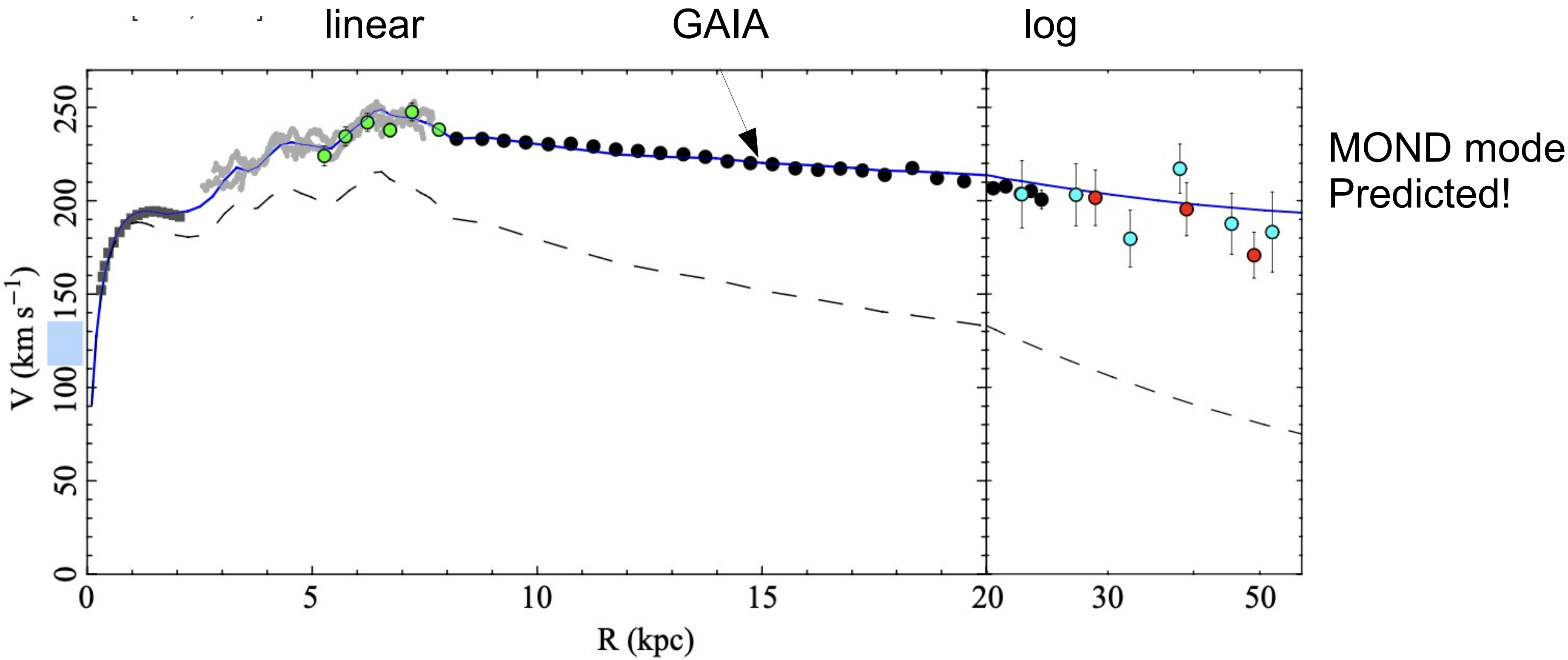
Many questions!

if so, with Modified inertia, MOND effects should show up in nongravitational phenomena. Can we think of feasible tests of this?

One needs to have a MONDian cosmology!

The Milky Way: rotation curve from various sources

(McGaugh 2023)

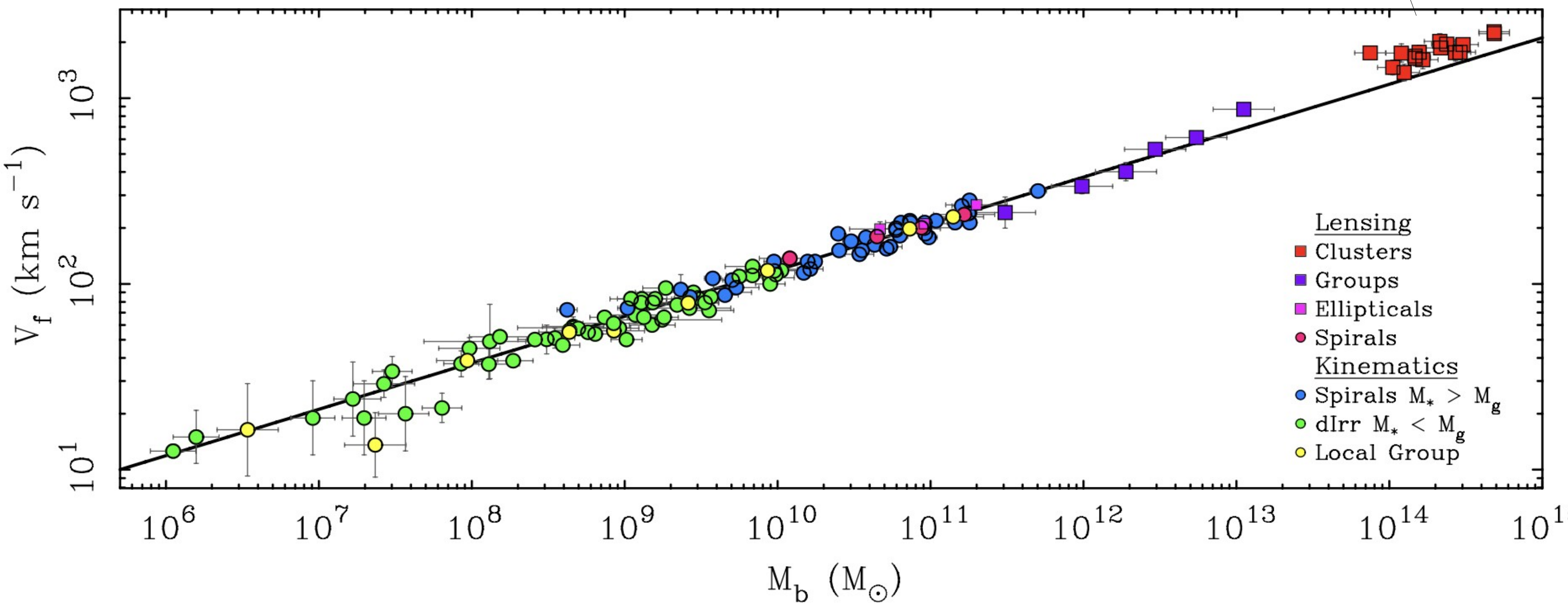


. The rotation curve of the Milky Way. Light gray points are the terminal velocity data (McClure-Griffiths & Dickey 2007, 2016) to which the model was fit (McGaugh 2018, green line). The model was not fit to the data of Portail et al. (2017) (dark gray squares at R < 2.2 kpc) or Eilers et al. (2019) (black squares), but is nicely consistent with them (McGaugh 2019). It also extrapolates well out to 50 kpc as traced by K giants (red squares) and BHB

Missing mass in galaxy clusters??
No, because stellar M/L has been underestimated
Zhang+2026, Phys.Rev.D.,V.113

From the MOND white paper

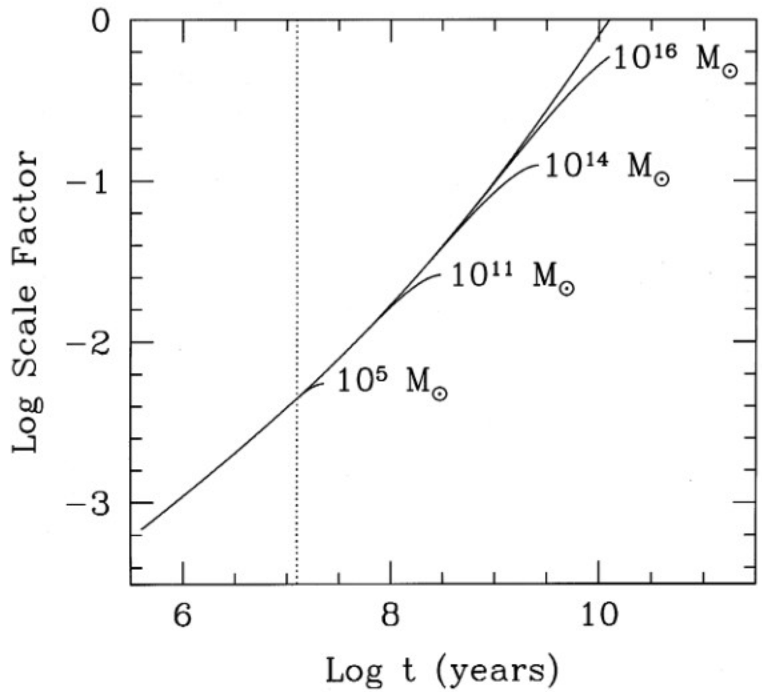
Relation between total baryonic mass
and asymptotic circular velocity over 9 orders of mag (Lelli+2026)



Cosmology ...en tramites

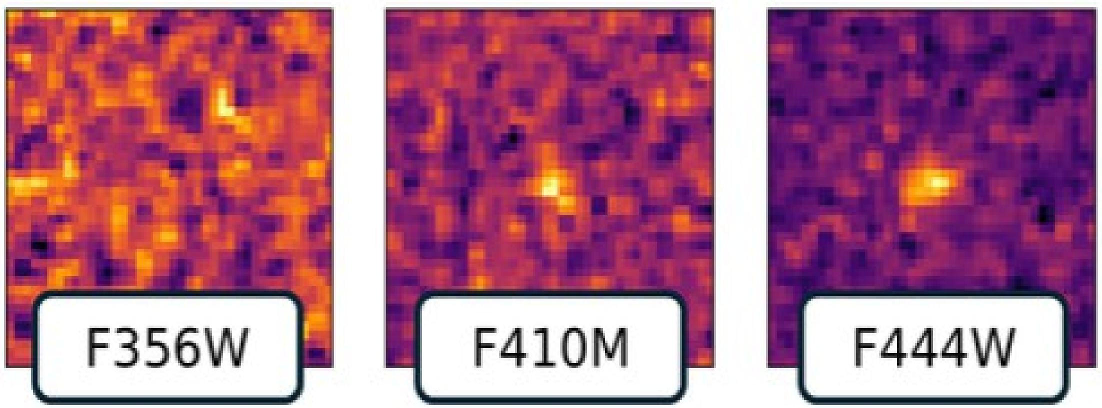
Cosmology with modified Newtonian dynamics (MOND),
Sanders1998, MNRAS 296,1009

MOND-force causes density fluctuations to quickly decouple from the Hubble-flow and recollapse



Massive galaxies form at
$Z \approx 40$
Should be virialized at
$Z \approx 15$

Mysteries of Capotauro: investigating the puzzling nature of an extreme F356W-dropout (Gandolfi+2025)



James Webb Space Telescope
NIRCam
NIRspec

- Brown dwarf?
- free floating exo-planet?
- dusty Balmer break galaxy z > 10?
- z=30 galaxy?
- strong line emitter galaxy

 not clear, but strong arguments for z=30

To take away

The MONDian phenomenology among galaxies is well established.

The predictive power of MOND is very strong!

MOND does not say anything about dark matter particles, but there is not much room left.

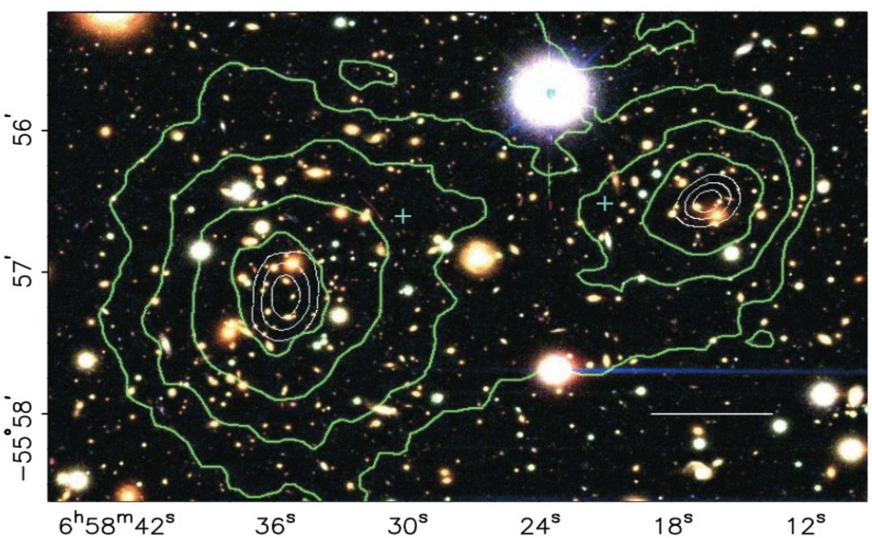
The "Bullet cluster" (1)

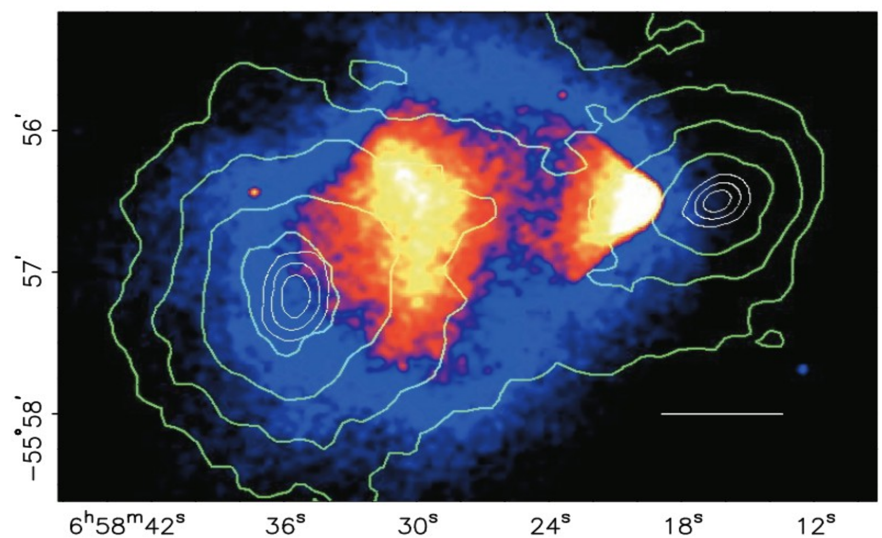
THE ASTROPHYSICAL JOURNAL, 648:L109–L113, 2006 September 10

A DIRECT EMPIRICAL PROOF OF THE EXISTENCE OF DARK MATTER[1]

DOUGLAS CLOWE,[2] MARUŠA BRADAČ,[3] ANTHONY H. GONZALEZ,[4] MAXIM MARKEVITCH,[5,6]
SCOTT W. RANDALL,[5] CHRISTINE JONES,[5] AND DENNIS ZARITSKY[2]
Received 2006 June 6; accepted 2006 August 3; published 2006 August 30

Z=0.269

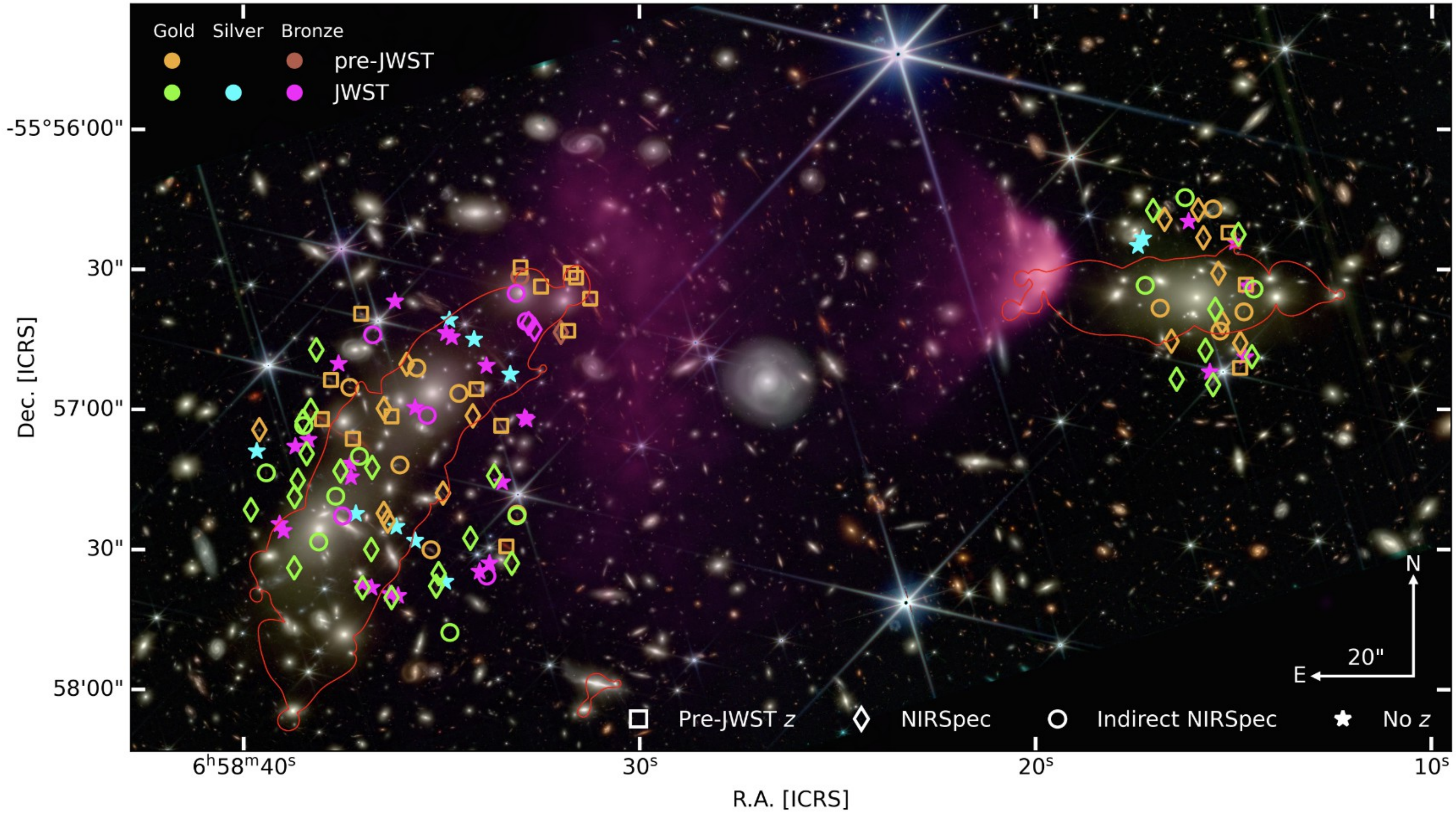




The "Bullet cluster" (2)

Updating the lens model with JWST data (new multiple images, new redshifts)

Rihtarsic+ 2026



The “bullet” cluster (3)

Hernandez 2026 – QUMOND
arXiv2604.10811

Rihtarsic+2026 - GR

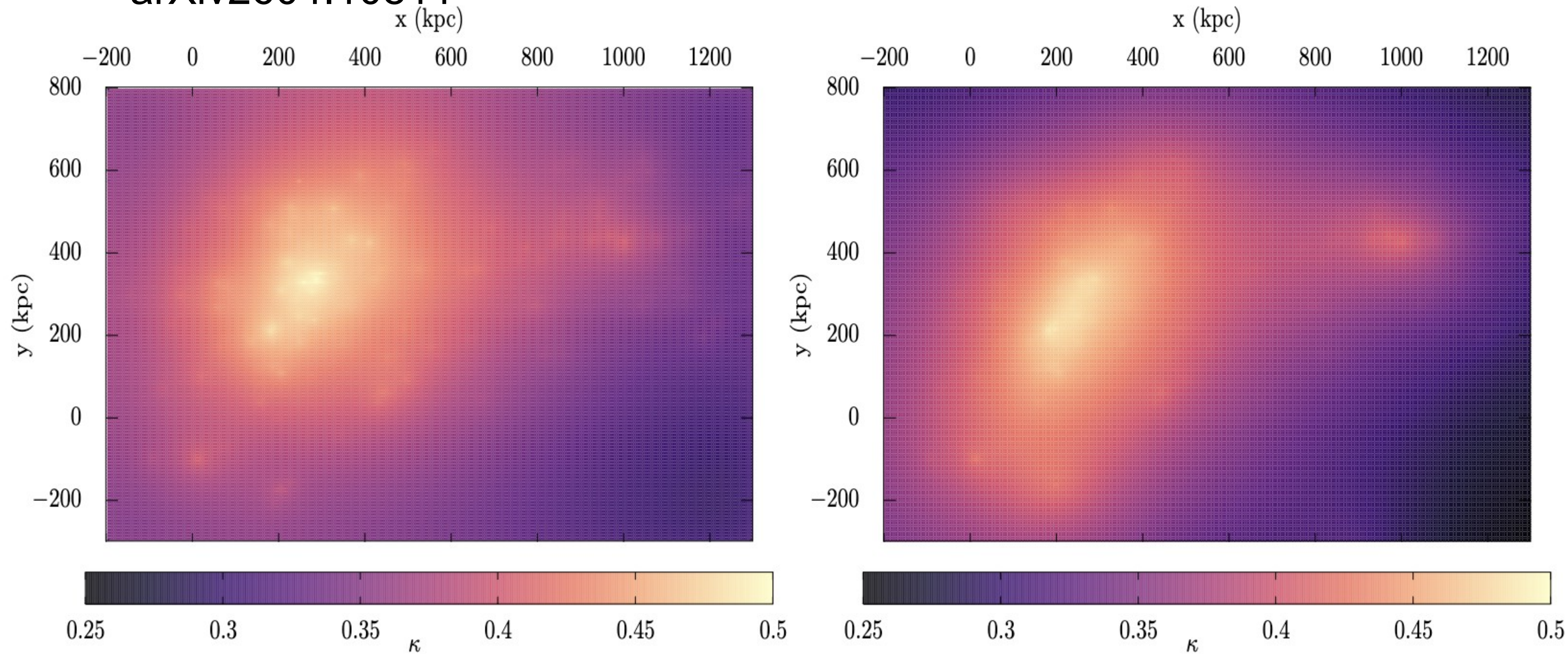


Figure 1. Left: This panel shows the total mass surface density projected onto the plane of the sky which QUMOND predicts will be inferred when interpreting the baryonic distribution of galaxies and X-ray gas of the Bullet Cluster within the framework of GR. Right:Total mass surface density on the plane of the sky as recently inferred by Rihtaršič et al. (2026) for the Bullet Cluster through GR lensing modelling. Both figures give mass surface density values in the same units of $1.826876 \times 10^9\ M_\odot \mathrm{kpc}^{-2}$, a close correspondence is evident.